Preface

> *«The fear of infinity is a kind of short-sightedness that destroys the possibility of seeing actual infinity, even if infinity in its highest expression created us and sustains us and through its secondary forms of transfinite surrounds us and even dwells in our minds».*
>
> (Cantor G., *Gesammelte Abhandlungen*, 1932)

The following reflections are related to a research study on mathematical infinity carried out over several years. Such a topic represented and still represents a fascinating subject matter constituting a primal interest for and involving scholars of different branches of knowledge. In particular, as far as mathematics and didactic mathematics are concerned, the issue of infinity has been considered from different perspectives and great attention has been paid to the most delicate historical moments, the epistemological obstacles specific to this topic and the related difficulties encountered by students, attending different educational levels, to approach the issue of infinity.

The innovative and charming viewpoint characterising the present research work, within the scope of the didactics of mathematics is to focus and investigate teachers' convictions on mathematical infinity. Firstly, we analysed primary school teachers' misconceptions supported by wrong mental images conditioning their convictions and also consequently their way of teaching. Subsequently, we concentrated on secondary school teachers' convictions and came to the conclusion that there are no great variations in the false beliefs revealed by teachers teaching in different educational levels.

The present work is formed of four chapters, all of them dealing with the issue of infinity as seen from different points of view and sharing a single common thread: didactics.

The first chapter provides readers with a chronological critical- historical outline in order to allow them to focus on fractures, non-continuities, radical changes in the evolution of a mathematical concept that underline the epistemological obstacles making infinity such a difficult topic to be conceived, accepted and finally learnt.

The second chapter offers a brief outline of those elements of didactic mathematics pertaining to the treatment of this thesis. In particular, we stated our approach, that is to be considered within the scope of the present Research in Didactics of Mathematics of the French School and whose attention is focused on the phenomenon of learning considered from a foundational point of view. In this respect, we will refer to what is intended today by *fundamental didactics* (Henry, 1991; D'Amore, 1999), i.e. everything concerning the basic elements of the research in mathematical didactics deriving from the various and complex analyses of the so-called "triangle of didactics": teacher, student and knowledge. In more detail, we will provide some useful hints for the interpretation of the analyses of the following chapters.

Chapters 3 and 4 describe the core of the research work. In the third chapter, primary school teachers' misconceptions on mathematical infinity are singled out by means of qualitative methodologies: analyses of questionnaire's collected answers and the related following discussion activities. The results have shown that infinity is, in general, an unknown concept, only managed by intuition and usually banally reduced to an extension of the finite.

These reflections revealed that the major difficulties related to the understanding of the concept of mathematical infinity are not exclusively due to epistemological obstacles, but are also strengthened and magnified by didactical obstacles. Obstacles deriving from the intuitive models provided by teachers to their students. Such models represent, without teachers being aware of it, real and proper misconceptions.

The same false beliefs have to be traced back in secondary school teachers' convictions who have been asked to analyse and discuss with the researcher their students' produced TEPs (D'Amore, Maier, 2002) concerning issues related to infinity and reported in chapter 4.

Our intention was to highlight how the object of our research has been so far underestimated, especially as a subject matter for training courses addressed to teachers. It is exactly this deficiency, in our opinion, that is one of the causes of the learning

problems encountered by secondary school students already possessing some previous convictions not suitable to face new cognitive situations.

Consequently, the fourth chapter is mainly based on the focus of our research, over the most recent years, that is to try to inhibit and therefore overcome those models turning into obstacles in teachers' minds and hence in turn in students' minds. The aim is to propose a learning pathway that envisages specific training courses for teachers that take into account the peculiar and intuitive aspects related to infinity, as well as the outcomes obtained by researchers in didactic mathematics. This kind of training will allow the participant teachers to properly deal with the concepts linked to infinite sets and even get their students involved in fruitful and meaningful activities in order for them to build intuitive images consistent with the theory of infinite sets.

Moreover, various present and future research studies have been introduced which we intend to carry out and that are particularly focused on teachers' and students' misconceptions on geometrical primitive entities surveyed from different points of view. This latter choice, outwardly distant from the world of infinity, actually derives from the acknowledgment that teachers' and students' misconceptions on geometrical infinity depend in most cases on those misconceptions regarding geometrical primitive entities.

The feeling pervading this dissertation is that this research work represents to the author just the beginning of a journey which has no end in sight and which is proving, year after year, to be ever more fruitful, challenging and involving.