

Report about the master thesis by Silvia Sbaragli “Teachers’ convictions on mathematical infinity”

The thesis consists of four chapters: chapter 1 sketches a historical overview of the notion of infinity in mathematics, chapter 2 is devoted to the theoretical framework of the present study, chapter 3 and 4 presents the questions, hypotheses and results of the various studies carried out by the author. The structure of the work is well organized although a conclusion would have allowed the author to point out the main results of her study.

Chapter 1 presents the concept of infinity through the works of great mathematicians. The study is well done and clearly expressed with several excerpts included in the text. The aim of this chapter is to “shed some light on the origin of epistemological obstacles” (according to the terms of the author) related to infinity. As for a future and further investigation of this aspect, it would be good to end the chapter with a section synthesizing the evolution of the concept over time and showing the various epistemological obstacles developed in this evolution. This will allow to relate chapter 1 with the following others.

Chapter 2 presents and comments some key concepts of didactics: the didactical contract, images and models, conflicts and misconceptions, the didactical triangle and epistemological obstacles. In general the presentation is well documented. The term “misconceptions” originating from the States is may be not the most appropriate for referring to the “incorrect” students’ knowledge. The notion of “correctness” is not absolute and always refers to a given knowledge, the reference knowledge which may also evolve. The criteria of rigor have been dramatically changing over time in mathematics. Any conception has a domain of validity and is efficient on this domain. If not, it will not survive. Any conception is partly incorrect and partly

correct. Therefore it seems to be more productive to speak about conceptions with a domain of validity and to try to determine what this domain is. The emphasis on the role of teachers in the didactical transposition, as presented in the study, is substantial. In the future the role of the *noosphere* in the process could be analyzed more in depth: designers of curricula, writers of textbooks...

Chapter 3 presents the questions, the hypotheses, the methodology in a first part well articulated. Then the results of the analysis of the collected data are presented. The text is essentially devoted to the analysis of the questionnaire and of the answers of teachers to questions of the researcher. This part is highly interesting. I wonder whether the notion of time could have been taken into account in the discussion of the teachers' conceptions. It seems that the potential infinity is strongly linked to a process developing over time and never finishing. ("everlasting process" in the words of the author p.81):

B: "I know infinity, it means to keep going on as with numbers ... for ever"

A: " Talking about the number, I show them the line with numbers and I say that they never end"

M: " I can keep on dividing always the same quantity"

This notion of time seems to belong to this basic metaphor grounding the mathematical notions as formulated by Lakoff and Nunez.

The investigation of the conceptions of infinity is related to the conceptions of segment and line in the rest of analysis, showing how conceptions may be shaped by the representations used to refer to mathematical abstract ideas. The author identifies the mental images developed by teachers about segment and line, relating them to the *necklace model* proposed by d'Amore and Arrigo. For a future treatment of the subject I would suggest to refer also to the work of Gonseth.

At the end of chapter 3, the author provides in section 3.4 the answers to the research questions. The answers are clear and well formulated. The answers to question 1 are

especially rich. As for future lines of research, it could also be interesting to consider a systematic collection of representations used by textbooks in addition of the answers to the questionnaire.

Chapter 4 is gathering old, present and future research of the author and is not as well structured as the preceding ones. It is mixing a teacher training course and results of investigations on students conceptions by using TEPs (p. 121). I think that that the author should have more developed the part devoted to the course. As for further reference, it would be interesting and useful to point out and describe in details the contents of the teachers' training course.

Briefly concluding, I would say that the choice of the mathematical notion of infinity in relation to epistemological obstacles and teachers' conceptions is excellent. As said in the thesis (p.61), infinity is an emblematic example of the notion of epistemological obstacle. The work investigates from two perspectives these obstacles, the historical one and the didactical one. This latter perspective addresses both teachers' and students' conceptions. It is a rich work based on a serious review of literature. The last chapter refers to a new research aspect that is still to be considered a work in progress and therefore needs further investigation, but on the whole my global opinion is very positive.

Grenoble, le 12 octobre 2004

Colette Laborde

Professeur des universités

IUFM de Grenoble et Université Joseph Fourier

46 av Félix Viallet

38 000 GRENOBLE

FRANCE