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Anti-Malthus: Conflict and the evolution of societies

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ABSTRACT

The Malthusian theory of evolution disregards a pervasive fact about human societies: they expand through conflict. When this is taken account of the long-run favors not a large population at the level of subsistence, nor yet institutions that maximize welfare or per capita output, but rather institutions that generate large amount of free resources and direct these towards state power. Free resources are the output available to society after deducting the payments necessary for subsistence and for the incentives needed to induce production, and the other claims to production such as transfer payments and resources absorbed by elites. We develop the evolutionary underpinnings of this model, and examine the implications for the evolution of societies in several applications. Since free resources are increasing both in per capita income and population, evolution will favor large rich societies. We will show how technological improvement can increase or decrease per capita output as well as increasing population.

1. Introduction

> no possible form of society [can] prevent the almost constant action of misery upon a great part of mankind

> There are some men, even in the highest rank, who are prevented from marrying by the idea of the expenses that they must retrench (Malthus, 1798)

The overall goal of this paper is to establish a theoretical setting of interacting societies in which it is conflict that determines long run success or failure. We identify assumptions under which “the strongest society wins” in the long-run, and examine the limitations and subtle implications of these assumptions. What will matter is willingness to expand and total resources which can be devoted to expansion – hence size matters. We attempt to build the theoretical setting in a way that can easily be applied to study practical problems of particular societies both contemporary and historical in order to understand which institutions are likely to be persistent. To illustrate this we examine several simple applications.

A key idea of the paper is that conflict resolution depends not only on the ability of players to influence their neighbors, but also on their desire to do so. Our main conclusion that with a single dimensional measure of strength the strongest society will be observed most of the time over the long run is rather intuitive. However, as those familiar with the evolutionary literature will appreciate to actually establish such a result in a clean form is not trivial. Moreover, not all implications of our assumptions are so obvious as the fact that the strongest society wins. Indeed, strictly speaking, the strongest society does not win. Rather it is the strongest incentive compatible arrangement that matters – non-Nash equilibria stand no chance in the long run, no matter how strong they might be. Second, it is not the strongest Nash

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equilibrium that wins. Societies can differ in their attitudes towards influencing neighbors. Societies, no matter how strong, that do not attempt to expand aggressively also will not survive in the long run. Rather it is the strongest incentive compatible and expansionary society that wins. Another point that is subtle is that expansionary attitudes are not important from the perspective of imposing particular institutions on neighbors – in fact the actual work of disrupting societies in the theory as well as in reality is by barbarian hordes – aggressive, powerful groups that however do not have especially durable institutions. Alexander the Great and Tamurlane come to mind in this context. Rather the importance of an aggressive expansionary posture is that when invaders achieve some success by conquering land the outward looking society aggressively attempts to recover the lost land, thus preventing gradual "whittling away" of territory.

The other key notion of the paper is the scalar measure of the ability of a society to disrupt neighbors or avoid disruption. The idea is best illustrated through a simple example. We are all familiar with the caricature of the Malthusian theory of population: population grows until it is checked by disease and starvation. In the long-run we are all at the boundary of subsistence, on the margin between life and death. And while we may seem to have escaped for a time, perhaps ultimately the rapidly growing developing countries will overwhelm the gradually shrinking rich developed world and sink us all back into misery. Malthus was more subtle in his thinking than this caricature: while he wrote of positive checks on population such as disease and starvation, he also wrote of preventative checks such as delayed marriage. Now let us take into account that societies do interact, and imagine two societies side by side. One is a society of unchecked breeders, of subsistence farmers living on the edge of starvation, their population limited only by the lack of any additional food to feed extra hungry mouths. Next door is a society with high property requirements for marriage and strong penalties for out-of-wedlock birth – a social arrangement quite common in history. This non-Malthusian society naturally has output well in excess of subsistence. Both social arrangements are incentive compatible. Who will dominate in the long-run? What happens when a disciplined and rich society turns its covetous eye towards the land of their more numerous but poorer neighbors? How indeed are the wretched poor – for whom to take even an hour away from toil in the fields is to starve – to be able to defend themselves from well-fed and well-armed intruders? The question answers itself. In this view *free resources* are the output available to society after the payments necessary for subsistence and for the incentives needed to induce production are made and after other claims such as transfer payments and resources absorbed by elites are paid. What matters for evolutionary purposes is the quantity of these free resources and the amount of them directed towards state power that can be used in conflict between societies.

We explore the consequences of the model in a series of examples. In the Malthusian model the theory gives a positive theory of population size: as long as there are incentive compatible institutions that control population growth, the equilibrium population is the one that maximizes total free resources. This is inconsistent with growing so large as to reach subsistence, as such a society generates no free resources. It is equally inconsistent with maximizing per capita output, since this requires a very tiny society that generates many free resources per person, but very few in total. Rather the long-run population is at an intermediate level, greater than that which maximizes per capita income, but less than subsistence.

We then examine the impact of technological change in a population setting and uncover very non-Malthusian results. Malthus predicts that the benefits of technological change will in the long-run be dissipated entirely in increased population with no increase in per capita output, which remains at subsistence. When there is relatively strong diminishing returns on plots of land, maximization of free resources implies that improved technology results primarily in increased per capita output. However, depending on the underlying returns to population size, technological change can also result in diminished per capita output in some parameter range. The Malthusian case of per capita output independent of technology will only occur as a non-generic accident. For simple and plausible cases, continued technological improvement first lowers then raises per capita output. This theory is very much more in accord with the evidence than Malthusian theory.[1]

Availability of free resources leads more broadly to a positive theory of the State: it has implications for institutions other than those that govern population size. It does not imply, as does, for example, the theory of Ely, economic efficiency.[2] Ely (2002) shows that if institutions spread through voluntary migration people will move to the more efficient locations and that in the long run this favors efficient institutions over inefficient ones. But we do not believe that historically people have generally moved from one location to another through a kind of voluntary immigration into the arms of welcoming neighbors. Rather people and institutions have more often spread through invasion – most often in the form of physical conquest, but also through means such as proselytizers and missionaries, or just exploration of new territory. In a setting of moral hazard, we show how inefficiently low levels of output (Section 7) can indeed arise.[3] We also use the example to explore in greater detail how individual choices can result in free resources or not.

The technical approach we take is the evolutionary one pioneered by Kandori et al. (1993) and Young (1993). Like the earlier literature we suppose that people adjust relatively rapidly to new circumstances. In that literature this was represented by what is often called the "deterministic" dynamics which is generally a variation on the best-response or

[1] This theory of population size of a given geographical extent should be compared to the theory of Alesina and Spolaore (2003) who examine the optimal geographical extent of a nation.

[2] Ely uses a model similar to the one used here, but similar results using more biologically oriented models have been around for some time. For example Aoki (1982) uses a migration model to study efficiency, while more recently Rogers et al. (2011) use a migration model to show how unequal resources can lead to long-run inequality.

[3] There are many other channels through which evolution can lead to inefficiency. For examples Bowles (2004) discusses how inefficiency can arise in a Kandori et al. (1993) and Young (1993) type of setting with groups when they are of different sizes or have different memory lengths.

replicator dynamic. Those deterministic dynamics suppose an adjustment process towards individually optimal strategies, and if they converge generally speaking the incentive constraints are satisfied and the point of convergence is a Nash equilibrium. However, as a reader of that literature might be aware, these dynamics are badly behaved in many games, and the earlier evolutionary literature focused on particular limited classes of games such as coordination games in which the deterministic dynamic is particularly well behaved. We do not think the misbehavior of the deterministic dynamic is especially interesting as people seem in fact to rapidly reach Nash equilibrium, and, as pointed out, for example, in Fudenberg and Levine (1998), the behavior of these dynamics when they do not converge is not especially plausible. As underlying model of "rational" individual behavior we take not these deterministic dynamics, but rather a simplified version of the stochastic dynamics developed more recently by Foster and Young (2003). This gives global convergence, at least in the stochastic sense, and enables us to give clean theorems without limiting attention to particular classes of games.

While the earlier literature supposed that the deterministic dynamic was perturbed by random mutations, we take the view that these small random changes – disruptions to existing arrangements if you like – are influenced instead by the relative strength of societies. Our strongest assumption is that this strength is measured by a single scalar quantity. We also assume that initially a tiny "invading" society has a negligible chance of disrupting existing social arrangement, but that once it becomes comparable in size to the pre-existing society the chances it is able to further disrupt the status quo become appreciable. Our approach is a variation on the conflict resolution function introduced by Hirshleifer (2001) and subsequently studied in the economic literature on conflict.[4]

The idea that evolution can lead to both cooperation and inefficiency is scarcely new, nor is the idea that evolutionary pressure may be driven by conflict. There is a long literature on group selection in evolution: there may be positive assortative matching as discussed by Bergstrom (2003). Or there can be noise that leads to a trade-off between incentive constraints and group welfare as in the work of Price (1970, 1972). Yet another approach is through differential extinction as in Boorman and Levitt (1973). Conflict, as opposed to migration, as a source of evolutionary pressure is examined in Bowles (2006), who shows how intergroup competition can lead to the evolution of altruism. Bowles et al. (2003) and Choi and Bowles (2007) study in group altruism versus out group hostility in a model driven by conflict. Rowthorn and Seabright (2010) explain a drop in welfare during the neolithic transition as arising from the greater difficulty of defending agricultural resources. As we indicate below, Bowles and Choi (2013)'s model of coevolution to explain the neolithic transition can be well understood in our framework.

More broadly, there is a great deal of work on the evolution of preferences as well as of institutions: for example Blume and Easley (1992), Dekel and Yilankaya (2007), Alger and Weibull (2010), Levine et al. (2011) or Bottazzi and Dindo (2011). Some of this work is focused more on biological evolution than social evolution. As Bisin (2001) and Bisin and Topa (2004) point out the two are not the same.

This paper is driven by somewhat different goals than earlier work. We are interested in an environment that can encompass relatively general games and strategy spaces; in an environment where individual incentives matter a great deal; and in an environment where the selection between the resulting equilibria is driven by conflict over resources (land). By employing the stochastic tools of by Kandori et al. (1993) and Young (1993) we are able with relatively weak assumptions to characterize stochastically stable states – the "typical" states of the system – as those among the incentive compatible states that feature large societies maximizing free resources.

2. The economic environment

Time lasts forever $t=1,\ldots$. There are J identical plots of land $j=1,\ldots J$. On each plot of land there are N players $i=1,\ldots,N$. In each period t each player i on each plot of land j chooses one of a finite number of actions $a_t^{ij}\in A^i$. Actions describe production, consumption, reproduction and political decisions. We use $a_t^j\in A$ and $a_t^{-ij}\in A^{-i}$ for profiles of actions on a particular plot of land j in period t.

Players care only about the actions taken by players living in the current period – they are myopic, which is to say we assume that periods are long enough to encompass the horizon of the players – and they care only about actions taken on the same plot of land on which they reside. Preferences of player i are described by a utility function $u^i(a_t^j)$. We refer to the game on a particular plot of land induced by these utility functions during a particular period as the *stage game*.

Of particular interest on each plot are the (pure) *Nash equilibria* of the stage game. These are the profiles a_t^j such that a_t^{ij} is a best-response to a_t^{-ij} for all j. There is of course no guarantee that pure strategy equilibria exist. However, as is standard, we may introduce a finite grid of mixed strategies and by doing so guarantee the existence of approximate equilibria. We can then weaken the behavioral assumption below so that approximate equilibria are absorbing or we may perturb payoffs a small amount to get exact equilibria. In this sense existence is not an important conceptual problem, and indeed we are interested not in the case where existence may be problematic, but the case, such as in repeated or social norm games, where there are many, many equilibria. To avoid any technical issue, we will subsequently assume existence.

Plots of land do interact with each other, but only through conflict. Interactions between plots, as well as behavior, are probabilistic and some consequences have negligible and other appreciable probability. To formalize this we introduce a

[4] See, for example, Garfinkel and Skaperdas (2007) or Hausken (2005). An important focus of this literature has been in figuring out how consumption shares are determined by conflict resolution function.

noise parameter $\mathcal{E} \geq 0$. Subsequently we will be considering limits as $\mathcal{E} \to 0$. Following the standard terminology of evolutionary theory, such as Young (1993), suppose that $Q[\mathcal{E}]$ is a function of $\mathcal{E}$. We say that Q is *regular* if $r[Q] \equiv \lim_{\mathcal{E}\to 0} \log Q[\mathcal{E}]/\log \mathcal{E}$ exists and $r[Q]=0$ implies $\lim_{\mathcal{E}\to 0} Q[\mathcal{E}] > 0$. For a regular Q we call $r[Q]$ the *resistance of Q*. Notice that a "lower probability" in the sense of a more rapid decrease as $\mathcal{E} \to 0$ means a higher resistance; by an *appreciable probability* we mean a resistance of zero. Otherwise we say that the probability is *negligible*.

Conflict is resolved through a conflict resolution function. Formally, depending on players play on the various plots, there is a possibility that each period t a single plot of land k is *disrupted* to an action profile $a^j_{t+1} \in A$ the following period. This disruption may have the form of conquest, that is the new profile that k is forced to play may be the same as that of a "conqueror" j, but it is a more general concept: for example, the result of conquest may not be that the conquered adopt the customs of the conquerors, but rather than the conquered fall into anarchy. Let $a_t = (a^j_t)^{j=1,\dots J}$ denote the profile of actions over players and plots. The probability that plot k is disrupted to action a^j_{t+1} (which it will play at $t+1$) is given by the *conflict resolution function* $\pi^k(a^j_{t+1}, a_t)[\mathcal{E}] \geq 0$ where since at most one plot can be disrupted $\sum_{k=1}^J \sum_{a^j_{t+1} \neq a^j_t} \pi^k(a^j_{t+1}, a_t)[\mathcal{E}] \leq 1$. We assume that this inequality is strict, so that there is a strictly positive probability that no disruption occurs, and that $\pi^k(a^j_{t+1}, a_t)[\mathcal{E}] > 0$ for all j when $\mathcal{E} > 0$. Notice in particular that the conflict resolution function depends on the noise parameter $\mathcal{E}$ and in particular admits negligible probabilities.

2.1. Histories and player behavior

The behavior of players depends on the history of past events as well as their incentives. Let H denote the set of L-length sequences of action profiles in all plots. At the beginning of a period the *state* is $s_t \in S \equiv H^1 \times \dots H^J \times \{0,1,2,\dots,J\} \times A$, that is a list of what has happened on each plot for the previous $L \geq 2$ periods $\tau = t-L+1, t-L+2, \dots t$, plus an indicator of which plot has been disrupted and the action to which it was disrupted. So an element s_t of the state space S has $J+2$ coordinates: the first J are histories of the actions, $s^j_t = h^j_t$, $j=1,\dots J$ where $h^j_t = (a^j_\tau)^t_{\tau=t-L+1}$; coordinate $s^{J+1}_t \in \{0,1,2,\dots J\}$ denotes the disrupted plot, where $s^{J+1}_t = 0$ is used to mean that no plot has been disrupted; and the last coordinate indicates the new action (if any), so $s^{J+2}_t \in A$. The stochastic process on which the paper is focused will be defined to be Markov on this state space, and we assume that there is a given initial condition s_1.

We now describe how the action profile on each plot j is determined at time t. If a plot was disrupted, that is $j = s^{J+1}_{t-1} > 0$, then players on that plot play $a^j_t = s^{J+2}_t$. Otherwise play is stochastic, each player plays independently, and play depends only on the history at that plot: we denote by $B^i(s^j_{t-1})$ the probability distribution over A^i played by player i at time t on plot j.

For each player we distinguish two types of states:

Definition 1. A *quiet state* s_t for player i on plot j is a state in which the action profiles have not changed on that plot, $a^j_{t-L+1} = a^j_{t-L+2} = \dots = a^j_t$, and for which a^{ij}_t is a best response to $a^{-i,j}_t$. We call a^{ij}_t the *status quo response*. Any state for player i on plot j other than a quiet state is a *noisy state*.

In other words, in a quiet state, nothing has changed and player i has been doing the "right thing" for at least L periods. In this case, we assume that if not disrupted, the player continues to play the same way; otherwise there is some chance of picking any other action:

Assumption 1. If s_{t-1} is a quiet state where a^{ij}_t is the status quo response, then $B^i(s^j_{t-1})(a^{ij}_t) = 1$. If s_{t-1} is a noisy state for player i on plot j then $B^i(s^j_{t-1})(a^{ij}_t) > 0$ for all $a^{ij}_t \in A^i$.

Notice that in a noisy state the probability of change is appreciable because it is positive and does not depend upon $\mathcal{E}$. This means that in a noisy state change is quite rapid until a quiet state is reached again. This will have the implication that Nash equilibrium is reached relatively rapidly following a disruption. This assumption captures the idea that even in changing times, while society as a whole may be disrupted, people manage to accommodate themselves to new circumstances and achieve incentive compatibility relatively quickly. For example, refugees during time of war may be quite miserable, but nevertheless generally seem to adjust in a sensible way to their new constraints. Similarly in prisoner of war camps, people seem to quickly adjust develop new stable institutions with a well organized hierarchy and trade – for example using cigarettes as currency.

Definition 2. A state s_t is a *Nash state* if every plot of land is in a Nash equilibrium and it is quiet for every player in every plot.

Notice that if a state is Nash then all plots are quiet, and hence unless there is a disruption, the next state will be the same as the current state. On the other hand a disrupted plot begins a possibly long epoch of turmoil which however, with positive probability, will end with the plot entering an existing society, which will then be strengthened. The process of evolution of societies is thus viewed as more flexible and general than a military conquest followed by submission of a loser. Societies are introduced formally in the next section.

Remark 1. This dynamic is a simplified version of Foster and Young (2003) – it is a simple and relatively plausible model. It has the implication that in the absence of conflict each plot will be absorbed in some Nash equilibrium, and that all of these equilibria have some chance of occurring.

3. Societies and conflict

We now wish to examine the conflict resolution function in greater detail. The central idea of the paper is that conflict resolution depends in an important way on two things: the ability of players to expand and their desire to do so.

The ability to expand depends on size: a prospective invader would find it much easier to conquer, say, Singapore, than, for example, Shanghai. The reason is that China, while per capita a poorer society than Singapore, has a much larger and more capable military. In other words, plots of land are organized into larger societies, and the ability of a society to defend itself – or to conquer other societies – depends at least in part on the aggregate resources of that society, not merely the resources of individual plots of land. To capture this idea we must specify how plots of land aggregate into larger societies. Since we require that behavior on a plot of land be governed by individual choices on the plot we want to assume that aggregation choice depends on the chosen profile. The question arises as how the desires of different plots are reconciled.

There are many complicated possibilities for plots to form alliances: one plot playing $a_t^j = A$ may be willing to ally only with plots playing B, while a plot playing B may be willing to ally with either A or C. As our goal is not to understand the details of coalition formation we simply assume that profiles are partitioned into *societies*, with the members of an element of the partition agreeing that they are willing to ally themselves with any other profile in the same subset. Formally we assign each action profile a_t^j an integer value $\chi(a_t^j)$ indicating which society that profile wishes to belong to, with the convention that $\chi(a_t^j) = 0$ indicates an unwillingness to belong to any larger society. All plots j with a common non-zero value x of $\chi(a_t^j)$ then belong to the corresponding society, which will then be represented by that integer x.

Notice that implicitly this requires that if a plot is willing to ally itself, it is willing to ally itself with plots using an identical action profile. Moreover, a plot that changes its profile may by doing so change societies. In the context of anonymous plots that are differentiated only by the action profiles of the individuals on those plots this seems a sensible simplifying assumption. Moreover, from the broad perspective of social behavior it makes sense the alliances are associated with similarity of culture: for example is it widely thought that the EU intervened in the Yugoslavian civil war because "Yugoslavia is a Western country" while not intervening in various African civil wars because of a lack of affinity with those countries. Similarly Islamic countries will generally support one another in conflicts with non-Islamic nations such as the conflict between Israel and Palestine. However, we do not rule out "multiculturalism", that is, a plot may agree to be allied in a single society with other plots that use different profile – the European Union springs to mind as an example of such a society. We discuss aggregation map χ in more detail in Section 5.

Societies not only vary in size, but are also differentiated by their inclination to export their ideas and social norms. Regardless of the form of expansion, expansionary institutions are not universal – an insular society is not likely to expand.[5] Religions such as Christianity and Islam have historically been expansionary trying actively to convert nonbelievers. By contrast since the diaspora Judaism has been relatively insular in this respect, and the same has been true of other groups such as the Old Believers in Czarist Russia. We have already denoted by $\chi(a_t^j) = 0$ *isolated* plots of land that are unwilling or unable to agree on belonging to a larger collectivity. We classify the remaining societies into two types: expansionary for those that actively attempt to spread themselves or non-expansionary for those that do not, and as a formal matter, since we require that the attitude of a plot of land reflects the underlying individual actions taken there, we use positive values of $\chi(a_t^j)$ for those societies that are expansionary, and negative values for those that are not.

Since we are interested in settings with many Nash equilibria, we assume that at least one Nash equilibrium is in fact expansionary:

Assumption 2. There is at least one stage game Nash equilibrium which is expansionary, that is has $\chi(a_t^j) > 0$.[6]

3.1. *Conflict resolution and state power*

We now come back to the "ability to expand" aspect mentioned above and introduce the notion of state power as a measure of ability to expand. We begin by describing how the organization of plots into societies and the actions taken on those plots results in the disruption of plots of land through conflict between different societies. This was represented formally by the conflict resolution function, now described in greater detail.

First we define the probability of society x being disrupted, denoted by $\Pi(x, a_t)[\mathcal{E}]$, as the probability that one of its plots is disrupted to an alternative action. Note the $\mathcal{E}$ parameter. In the case $x \neq 0$ this is given by

$$\Pi(x, a_t)[\mathcal{E}] = \sum_{k|\chi(a_t^k) = x} \sum_{a_{t+1}^j \neq a_t^k} \pi^k(a_{t+1}^j, a_t)[\mathcal{E}],$$

[5] Our notion of expansionism is connected to Aoki et al. (2011)'s theory of the transmission of innovations.

[6] Note that whether or not a society is expansionary plays no role in the determination of Nash equilibrium.

and for an isolated society playing a_t^k by

$$\Pi(a_t^k, a_t)[\mathcal{E}] = \sum_{a_{t+1}^j \neq a_t^k} \pi^k(a_{t+1}^j, a_t)[\mathcal{E}].$$

We make the technical assumption that the disruption function $\Pi(x, a_t)[\mathcal{E}]$ is regular and that resistance is bounded above. Without loss of generality we may take the upper bound on resistance to be one so that $r[\Pi(x, a_t)] \leq 1$.

As we said, the ability to expand depends not only on the desire to do so, but also on the resources available. Specifically we assume that the action profile in a plot generates a strictly positive value $f(a_t^j) > 0$ called *state power*. This has for the moment no economic content, but we ask the reader to interpret it as a scalar measure of the ability to disrupt neighbors and avoid disruption; concrete specifications of this function are deferred to later sections. What matters, however, in resolving conflict is not merely state power on a particular plot of land but rather the aggregate state power available to a society. For a non-isolated society $x \neq 0$ this is[7]

$$F(x, a_t) = \sum_{\chi(a_t^j) = x} f(a_t^j).$$

Note that if a society x is not present in a_t then the corresponding aggregate state power F is zero. Notice also that due to multiculturalism, a society's state power depends non-trivially on a_t because the admitted profiles will have different levels of state power, and the total depends on how many of each kind there are.

3.2. Disruption, expansionism and state power

We are now in a position to state our three assumptions relating the disruption probability Π to state power. The basic idea is that the more state power a society has the more disruptive it is to its neighbors and the less likely it is to be disrupted by its neighbors. Moreover, non-expansionary societies are not disruptive to their neighbors. We capture these ideas through a number of specific assumptions.

The first assumption is that comparing two societies, resistance to disruption is lower for the one with less state power, and indeed resistance to disruption when there is an expansionary society with at least as much state power is zero. Let $E(x)$ denote whether x is expansionary or not, that is, $E=1$ if $x > 0$, and $E=0$ otherwise.

Assumption 3 (*Monotonicity*). If $F(x, a_t) \leq F(x', a_t)$ then $r[\Pi(x, a_t)] \leq r[\Pi(x', a_t)]$, and $r[\Pi(x, a_t)] = 0$ if $E(x') = 1$. Moreover, if a_{t+1} differs from a_t solely in that society x has lost a single plot of land, then $r[\Pi(x, a_{t+1})] \leq r[\Pi(x, a_t)]$.

The first part says that if two societies coexist in the sense that they are part of the same a_t then the one with greater state power has at least the same resistance as the one with less state power. The second part strengthens this to say that an expansionary society with at least as much state power as a rival in fact has an appreciable chance of disrupting it. This rules out the possibility of there simultaneously being multiple expansionary societies for a substantial length of time, and enables us to use an analysis akin to Ellison (2000)'s method of the radius. Without it, the analysis is more akin to his method of the co-radius, and we have neither been able to establish the result nor provide a counter-example in that case. The third part says that losing land does not increase resistance.

Our next assumption on Π specifies that resistance depends only on the ratio of state power when there are only two societies. Say that a_t is *binary* if there are only two societies, which we denote as x and x'.

Assumption 4 (*Ratio*). If a_t is binary then

$$r[\Pi(x, a_t)] = q(F(x', a_t)/F(x, a_t), E(x')),$$

where q is non-increasing and left continuous in the first argument, $q(0, E) = q(\phi, 0) = 1$ and there exists $\phi > 0$ such that $q(\phi, 1) > 0$.

In other words, resistance in the binary case depends monotonically on state power and whether or not the rival society is expansionary. Moreover $q(0, E) = 1$ says that when the opponent has no state power resistance is at the highest possible level – recall that we have assumed that resistance is always bounded above by one. In addition $q(\phi, 0) = 1$ asserts that a plot that is not expansionary always generates the same maximal resistance regardless of how much state power it has available. Notice that the assumption $q(0, E) = 1$ applies to *mutations* – actions that are not currently being used. In this setup the chance of a mutation entering the population is the same (in resistance terms) for all mutations – the state power associated with the mutant action profile becomes available for initiating or defending against disruption only after it enters the population – that is, the period after the mutation takes place. This follows from our assumption that the societies corresponding to action profiles that are not currently in use have no state power. The idea is that mutants need a period to get organized.

[7] It may be that aggregate free resources grow less than linearly with the number of plots. For example two plots each with a unit of free resources may be weaker than a single plot with two units of free resources if not all the units can be mobilized for joint operations or there are other coordination problems between the plots. The earlier working paper version of this paper showed that the results here remain unchanged if linear aggregation is replaced with a non-linear aggregation provided that aggregate free resources for a society are strictly increasing in the free resources on individual plots.

Observe that Assumption 3 implies that $\overline{\phi} = \inf \{\phi | q(\phi, 1) = 0\} \le 1$, since eventually if an expansionary society has enough state power, it has an appreciable chance of disrupting a rival plot of land. Note that because $r[q(\phi, 1)]$ is left rather than right continuous we must use the inf here, and because we have assumed explicitly that there is some value of $\phi > 0$ for which the resistance is strictly positive, we know that $\overline{\phi} > 0$. Looking at what this means in terms of probability, we see that this zero up to $\overline{\phi}$ after which it becomes strictly positive. That is, in the limiting case a sufficiently small society has no chance at all for disrupting a plot from a larger one.

The last assumption on Π states that disruption is not more likely when opponents are divided. Let $\Upsilon(a_t)$ denote all the societies in a_t, that is the values of $x \neq 0$ in the range of χ plus the different values of a_t^j that correspond to isolated societies, that is with $\chi(a_t^j) = 0$.

Assumption 5 (*Divided Opponents*). If a_t is binary, $\tilde{a}_t$ has $F(x, a_t) = F(x, \tilde{a}_t)$ and $\sum_{x' \in \Upsilon(a_t) \setminus x} F(x', a_t) \ge \sum_{x' \in \Upsilon(\tilde{a}_t) \setminus x} F(x', \tilde{a}_t)$ then $r[\Pi(x, a_t)] \le r[\Pi(x, \tilde{a}_t)]$.

4. Dynamics and stochastically stable states

The dynamics of the stage game and of disruption together with the behavioral rules of the players induce a Markov process $M(\mathcal{E}, J)$ on the state space S defined in Section 2.1. We are interested in this process, but primarily in the limit of this process as $\mathcal{E} \to 0$.

Theorem 1. *For $\mathcal{E} > 0$ the process $M[\mathcal{E}, J]$ is aperiodic and irreducible and hence has a unique invariant distribution $\mu[\mathcal{E}, J]$.*

Proof. This follows from the fact that every combination of actions on every plot has positive probability. □

We denote by $S[0, J]$ the ergodic classes of $M[0, J]$.

Proposition 1. *$\sigma \in S[0, J]$ if and only if: (i) σ is a singleton, that is, $\sigma = \{s_t\}$, (ii) s_t is a Nash state, and (iii) s_t has either no expansionary society, or a single expansionary society such that all other societies (if any) have positive resistance to disruption.*

Proof. Follows directly from the definitions. See Appendix A.

Hereafter we simply write $s_t \in S[0, J]$. Recall that Nash states are quiet on every plot, that is on each plot there is a Nash equilibrium which has been played for at least L periods; in particular a Nash state assigns a single Nash equilibrium profile to each plot.

By Proposition 1 there are three types of Nash states in $S[0, J]$. There are *monolithic expansionary states* consisting of a single expansionary society; there are *mixed states* consisting of a single expansionary society and at least one non-expansionary society, and there are *non-expansionary states* in which there is no expansionary society.

We use the following Theorem from Young (1993):

Theorem 2. *$m = \lim_{\mathcal{E} \to 0} \mu[\mathcal{E}, J]$ exists and $m(s_t, J) > 0$ implies $s_t \in S[0, J]$.*

Let $S[m, J] \subseteq S[0, J]$ to be the set of states that have positive probability in the limit (that is $s_t \in S[m, J]$ iff $m(s_t, J) > 0$). These are called the *stochastically stable states*. Our main result characterizes these states. To do so we must consider monolithic expansionary states in more detail.

Recall that societies are integers, in particular expansionary societies are positive integers. Since there are finitely many profiles not all integers are in the image of the χ map. For positive x in the image of χ consider the set $\chi^{-1}(x)$ of profiles a_t^j which map to x. Then x can contain any combination of these profiles. So for any expansionary society x there will be some collection – empty if $\chi^{-1}(x)$ is empty – of corresponding monolithic expansionary states $S(x) \subset S[0, J]$, which correspond to different combinations of Nash states with profiles $\chi^{-1}(x)$ allowed by that society. As already mentioned, these different profiles may have different levels of state power. Let $f(x)$ denote the least average per plot state power in any of these states (or zero if $S(x)$ is empty). It is obvious but useful to point out for later reference that this minimum is achieved when all plots play profiles generating the least state power. We say that x is a *strongest expansionary society* if $f(x) = \max_{x' > 0} f(x')$. Note by Assumption 2 and the assumption that state power is strictly positive there is indeed at least one strongest expansionary society.

We can also extend the notion of a stochastically stable state to that of a *stochastically stable society*. This is a society for which all the corresponding monolithic states $S(x)$ are stochastically stable. The central result of the paper is

Theorem 3 (*Main Theorem*). *If x is a strongest expansionary society then it is stochastically stable. As to the converse: For J large enough every stochastically stable state $s_t \in S(x)$ for some strongest expansionary society x.*

Proof. Follows from least resistance tree arguments detailed in Theorem 5 and Corollary 3 in Appendix A.

Remark 2. Notice that in order to prove the converse we require a large number J of plots. The reason for this is simple: what matters is how the resistance of different states compare with each other. When J is small there may be ties. For example, with just two plots, any state is destabilized by a single mutation, regardless of the presence of state power or expansionism. Similarly, with a modest number of plots, while two different states with different levels of state power will have different thresholds that an invader will have to overcome in order to destabilize that state, the actual number of plots that the invader has to conquer may be exactly the same. However, no matter how small the difference in state power between two different states, once there are sufficiently many plots the different thresholds imply that the invader must disrupt strictly more plots to destabilize the stronger of the two states. Roughly, what a large number of plots insures is that differences between societies are not lost in the coarseness of the grid.

Remark 3. There are several important features of the Theorem for large J. First only monolithic expansionary societies are stochastically stable. Second the strength of a monolithic expansionary society is measured by the Nash equilibrium consistent with belonging to that society which has the least state power – the strength of a society is measured by its weakest member. Finally, among these incentive compatible expansionary arrangements it is having a weakest member with the most state power that counts. Notice also that there may be non-expansionary societies with much greater state power than any expansionary society, or expansionary societies with incentive compatible arrangements with much greater state power than $\max_{x'>0} f(x')$. Nevertheless these societies are not stochastically stable and in the long-run will not be much observed.

It is worth indicating how the stochastically stable states relate to the dynamics of the Markov process for $\epsilon > 0$. It is important to understand that the system does not in any sense converge asymptotically to the stochastically stable state. Rather the expected length of time the system spends at that state is roughly proportional to $1/\epsilon$ raised to the power of the least resistance of leaving the state.[8] The system is genuinely random: disruptions can and do occur. Suppose the system is currently in a stochastically stable state. Sooner or later there will be enough unlucky coincidences to disrupt it and the system will fluctuate randomly for some period of time as there is an appreciable probability that individuals will change their behavior. Eventually the system will settle down to some other steady state, not necessarily the stochastically stable one. However that steady state will also eventually be disrupted, more fluctuations will occur, then another steady state will be reached. At some point another stochastically stable state will be reached. The key point is that the amount of time spent at steady states is high relative to the amount of time the system spends fluctuating randomly, and the amount of time spent at the stochastically stable states is high relative to the amount of time spent during fluctuations and at steady states that are not stochastically stable.

Dynamic considerations also explain why we can use a very weak notion of expansionism. We do not assume that a disrupted plot is "conquered" and absorbed by the disrupting society as an immediate consequence of disruption. Rather disruption itself is enough to result in conquest in the longer run. Once a plot is disrupted it finds itself in a non-quiet state and goes through a period of change until it is absorbed in a Nash state. But if when it does so it fails to join with its stronger expansionary neighbor, it will be disrupted again. This process will repeat until it is eventually absorbed by this neighbor. This is all that is required in the proof of the theorem. Consequently an important message from the theory is that in the long run it is not conquering power that counts, but stability in the sense of resistance to disruption.

Remark 4. (*Relation to Literature on Group Evolution*) The novelty of our approach lies in the fact that we study group evolution as evolution of Nash equilibria. Existing literature in the area mainly focuses on the interplay between individual and group evolutionary selection: individual behavior which increases fitness of a group, typically some form of "generosity", may be harmful for individual fitness. This is the case both in the Haystack Model as in Maynard Smith (1982) or Richerson and Boyd (2001) and in Bowles' (2009) model of conflict and evolution. The equilibrium dimension in the group selection literature is generally missing. One exception is Boyd and Richerson (1990) who consider a setting with multiple Evolutionary Stable Strategies and show that group selection can be operative at the level of the equilibrium.

In relation to this trade-off our result may be interpreted as saying that evolution, favoring expansionism, favors generosity, which may be seen as a necessary condition for expansionism; but also that given generosity, it favors large groups maximizing state power, which is needed to survive competition between groups.

5. Social norm games and state power

In institutional design settings – such as repeated games – there are typically a plethora of incentive compatible outcomes. The evolutionary theory here is a theory in which the incentive compatible outcome that is the strongest expansionary one is most likely to be observed in the long-run. We now wish to examine concretely what that means. From a game theoretic point of view, modeling the fact that there are many social norms is no longer an open problem. The folk theorem points to the existence of many social norms. Although the basic theorem involves an infinitely repeated game with discounting there are folk theorems for games with overlapping generations of players as in Kandori (1992), for finite horizon games where the stage game has multiple Nash equilibria as in Benoit and Vijay (1987), and for one-shot self-referential games as in Levine and Pesendorfer (2007). As this literature is well developed we will adopt a simple two stage approach to get at the issues of the formation of societies and state power.

We are given an arbitrary finite *base game* with strategy spaces $\tilde{A}^i$ and utility functions $\tilde{u}^i(\tilde{a}_t^j) \geq 0$, where the non-negativity of payoffs is a convenient normalization. These actions represent ordinary economic actions: production, consumption, reproduction decisions and so forth. We are also given a finite list O of integers representing different types of societies. We will detail the connection between these types of societies and the map $\chi(a_t^j)$ after we describe the game itself.

[8] This is shown by Ellison (2000) who refers to this least resistance as the radius of the state.

We now define a two stage game. In the first stage each player chooses a base action $\tilde{a}_t^{ij}$ and casts a vote $o_t^{ij} \in O$ for participation in a particular society. Players have preferences $\tilde{u}^i(o_t^j) \geq 0$. We assume preferences are additively separable between payoffs in the base game and preferences over votes. While this is a useful simplifying assumption separating as it does economic decisions and decisions over what sort of broader society to belong to, situations in which it does not hold can be of interest. Consider for example the case of Switzerland. Here there is a substantial expenditure on defense including a requirement of universal military service and enrollment in the military reserves. One reason people are willing to participate is because of an implicit promise that Switzerland is non-expansionary: military forces will be used only to defend Switzerland and not sent abroad. It is easy to imagine a connection here between the utility from the economic decision – to provide substantial state power in the form of military expenditures – and the social decision – to be non-expansionary. A vote for an expansionary society might well be more attractive when combined with an all-volunteer military such as that in the United States. Similarly economic decisions over careers might well influence preferences over which type of society to belong to: a career soldier might as a consequence have a preference against being expansionary as he will have to bear the cost of the overseas fighting.

We turn now to the second stage of the game. As a consequence of first stage decisions there is a publicly observed state variable θ_t^j in a finite set of signals Θ. The probabilities of these signals are given by $\rho(\theta_t^j | \tilde{a}_t^j, o_t^j)$.

In the second stage of the game players have an option to punish other players by "shunning them." These choices may be based (only) on the public signal from the first stage. In particular in the second stage each player chooses an $N-1$ vector of 0's and 1's where 0 is interpreted as "do not shun" the corresponding opponent and 1 is interpreted as "shun" the corresponding opponent. We are also given a threshold $N-1 \geq N_1 > 0$. Any player who is shunned by N_1 or more opponents receives a utility penalty of $-\Pi^i$. There is no cost of shunning. As is the case with social voting, the utility penalty is additively separable with economic decisions: the penalty is simply subtracted from other payoffs. As in the case of social voting there may be situations in which there is an interaction between first stage interactive decisions and shunning or a cost of shunning. For example if you or I were to marry a child we had adopted we would probably be shunned. However to shun Woody Allen for this behavior is costly because he is an immensely talented film-maker and because he has made the economic decision to devote a great deal of time and effort to film-making. If he chose not to make films it would be much less costly to shun him.

Notice that the second stage game is constructed to be a coordination game: in particular for any subset of players it is a Nash equilibrium for all players to shun exactly the players in that subset. These equilibria are not terribly robust – for example to the introduction of costs of shunning – but there are many more robust – albeit more complicated models – such as having an infinite sequence of punishment rounds or the self-referential model of Levine and Pesendorfer (2007). Since the robustness plays no role here for expositional simplicity we use the simple model of costless punishment.

Stage game strategies now consist of a triple of $\tilde{a}_t^{ij}, o_t^{ij}$ and a map $m : \Theta \to \{0,1\}^{N-1}$. Following our previous notation, the space of these strategies is denoted by A^i. Payoffs are the expected value of the sum of $\tilde{u}^i(\tilde{a}_t^j) + \tilde{u}^i(o_t^j)$ and the cost of being shunned. We assume that after the game is complete – that is, after the shunning decisions are made – that these strategies become publicly known.

We now need to describe the relation between the social decisions o_t^{ij} and the map $\chi(a_t^j)$, that is, how do these votes translate into concrete decisions concerning societies and alliances? First we describe O in greater detail. The integer 0 represents an isolated society. The integer $+1$ is interpreted as being expansionary and willing to affiliate with any plot that uses exactly the same action profile a_t^j (action, punishment *and* voting). The integer -1 means non-expansionary and affiliate with any plot that uses exactly the same action profile a_t^j.[9] The remaining integers k are described by subsets $A_k \subset A$ representing different profiles that are acceptable to that society, with positive integers representing expansionary and negative integers representing non-expansionary. Note that some of these may be vacuous: for example some may be like Groucho Marx and accept as members of the society only plots that voted against joining. Obviously such vacuous society types do not matter. More interesting possibilities are subsets of the form $a_t^j \in A_k$ if and only if the corresponding $\tilde{a}_t^j$ has a specified value. Such a society admits members based only on the first period base game profile: behavior with respect to voting and shunning disregarded.

Notice that corresponding to the choices $+1, -1$ are many societies: those choices mean "exactly those plots identical to me belong to my society" so each action profile a_t^j represents a potentially different society. We wish to assign numerical indices to these different "exclusive" societies. To avoid conflict with societies $k \in O$ we do so by assigning each profile $a_t^j \in A$ a unique positive integer $\tilde{\chi}(a_t^j)$ larger in absolute value than the greatest absolute element of O.

The voting procedure presupposes a threshold $N_2 > N/2$. The $\chi(a_t^j)$ map is then defined as follows. If in a_t^j no element of O receives N_2 or more votes then the society is isolated and $\chi(a_t^j) = 0$. If $+1$ (the "exclusive" society corresponding to the action profile a_t^j) receives N_2 or more votes then $\chi(a_t^j) = \tilde{\chi}(a_t^j)$, that is the society is expansionary and admits as members exactly plots that play a_t^j. If -1 receives N_2 or more votes then $\chi(a_t^j) = -\tilde{\chi}(a_t^j)$, these being "exclusive" non-expansionary societies. Finally if $k \notin \{-1, 0, 1\}$ receives N_2 or more votes and $a^j \in A_k$ then $\chi(a_t^j) = k$. Otherwise the society is isolated – for example because it voted to join a society that will not admit it.

[9] The reason for this slightly convoluted approach to affiliation with a plot that uses the same profile including voting is that the size of the action space A^i depends on the size of O hence trying to put a list of elements of A^i in O is circular.

The idea of voting over what society to belong to may seem strange. In fact genuine voting decisions do take place, as for example, referendums on independence in places such as Quebec, Scotland and the South Sudan. However, voting here should be understood in the broader sense of “willingness to participate in the broader society.” So when Poland surrendered to Germany at the beginning of World War II implicitly the people of Poland – directed by the leaders who surrendered – agreed to belong to the German collective. Here also this was associated with practical decisions as well as “voting” – the effective agreement was to stop fighting with the Germans and follow German law, obey German police and so forth.

5.1. Perfect observability

We begin with the simplest case to study, that of perfect observability where θ_t^j perfectly reveals the first stage choices; we consider private information in Section 7. We assume that for all $\tilde{a}_t^j, o_t^j, i$ we have $\tilde{u}^i(\tilde{a}_t^j) + \tilde{\tilde{u}}^i(o_t^j) - \Pi^i < 0$ so that the punishment from being shunned overrides any possible gain from first period misbehavior. We then have the following folk theorem:

Proposition 2. *For any $\tilde{a}_t^j, o_t^j$ there exists a m_t^j such that $\tilde{a}_t^j, o_t^j, m_t^j$ is a Nash equilibrium.*

Proof. Take the map m_t^j so that any player i who fails to play $\tilde{a}_t^{ij}, o_t^{ij}$ is shunned by all opponents in the second stage, any player who does play $\tilde{a}_t^{ij}, o_t^{ij}$ is not shunned by any opponents. The payoff assumption shows then that playing $\tilde{a}_t^{ij}, o_t^{ij}$ is incentive compatible.

Next we wish to examine the role of state power in greater detail. We assume that free resources depend only on the base game profile $\tilde{a}_t^j$ and not on either voting or shunning.

Corollary 1. *If $+1 \in O$ then for large J stochastically stable states have per plot state power* $\max_{\tilde{a}_t^j} f(\tilde{a}_t^j)$.

Proof. Pick an element $\hat{a}_t^j$ of $\arg\max_{\tilde{a}_t^j} f(\tilde{a}_t^j)$, let o_t^j be a unanimous vote for $+1$ and let a_j^t be some corresponding Nash equilibrium strategy from Proposition 2. Then by Theorem 5 the society $\tilde{\chi}(a_t^j)$ has the greatest possible state power and so is stochastically stable. This implies also that for large enough J any stochastically stable society must have this same amount of state power.

5.2. Jewelry versus swords

Expansion may have many forms and motivations: two examples are conquest through warfare or conversion, but others are the desire to explore new territory, contact other societies and mix with them, propose values and possibly learn from outside communities. The Roman empire is a strong example of the first type of expansion; more modern expansions have often involved religious conversion – for example, the sending of religious missionaries, although this has often occurred in the context of warfare, for example the conversion of the South and Central American Indian populations to Christianity through a combination of conquest and missionary activity. Equally relevant is influence through exchange of goods spurred by explorations (think of Marco Polo), or the more modern culture spreading through the sale of goods ranging from Coca-Cola to television sets. Or going to the other extreme, we may think of the “curiosity”, that is the expansionism, of the primitive hunters–gatherers.

While there can be many forms of strength, in general in order to disrupt neighbors or defend against disruption it is generally important to have resources that are not being use for other purposes: we label these “free” resources. However, it is not enough that resource be free: they must also be used for military purposes, that is, for state power. We can see this idea in a simple example.

Suppose that a representative individual i produces “excess output,” or free resources of $\varphi > 0$ of a fixed amount above what is needed for subsistence, but must choose whether to take their share of “excess” output as jewelry or as swords, the latter representing state power. We take $\tilde{A}^i = \{0, 1\}$ where 1 means consume jewelry and 0 means consume swords. Utility is $\tilde{u}^i(\tilde{a}_t^{ij}) = \tilde{a}_t^{ij}$ so that people like to consume jewelry but not swords. On the other hand, jewelry is not of much use in conflict – a society armed with swords will quickly prevail over one armed with jewelry. Hence we take state power to be $f(\tilde{a}_t^{ij}) = \varphi(1 - \tilde{a}_t^{ij})$, that is swords only.

Technically there is not much to be learned from this formulation: state power is maximized by choosing swords over jewelry. Provided that this can be sustained as an equilibrium by the shunning technology the society that will be stochastically stable and most observed in the long-run will be the society that chooses swords over jewelry. This may appear to an observer to be dysfunctional in the sense that people like jewelry but not swords. Naturally the folk theorem allows many inefficient equilibria of this type, and we generally as economists do not pay much attention to them. But here the swords protect society from outside threats, so while inefficient this equilibrium is very functional indeed.[10]

[10] If jewelry that are consumed during peacetime may easily be given up and converted to swords in time of war then jewelry would be part of state power.

In the remainder of the paper we shall assume that there are effective institutions for converting free resources into state power, and so state power will be taken to be equal to free resources, and evolution will favor free resource maximization. Since free resources are a residual after necessary payments are made, free resource maximization is a lot like profit maximization and indeed our formal models have a strong similarity with models of profit maximization. Moreover, from an empirical standpoint, this connection may explain the historical importance of monarchies that can only be described as profit maximizing. But as the jewelry and swords example makes clear, this connection is not perfect: the residual – profits if you will – may be turned to many uses and only some of these uses – swords not jewelry – constitute state power. Hence a profit maximizing monarchy that through social norms is bound to use its profit for fighting and conflict is the type we expect to survive: a profit maximizing monarchy that spends its profits and large and beautiful palaces has less of a future.

6. Free resources in a Malthusian game

We now turn our model towards the study of population using a simple Malthusian type of example in which players are families who choose the size of their families. We assume that free resources are used entirely for state power so focus on the provision of free resources. We will contrast the Malthusian case where population size is chosen so that income per capita is at subsistence level with the case where it maximizes free resources, and explore how population and income per capita vary as technology improves.

We will model these economies as social norm games of perfect information, so there will be many Nash equilibrium population sizes: some large in which everyone is at the Malthusian level of subsistence, and others smaller in which output per capita exceeds that needed to survive and reproduce. Notice that Malthus recognized that there can be incentive compatible social arrangements that stabilize the population at a low level. In real societies, long before the advent of birth control, population was controlled – largely, of course, by abstinence from intercourse. It is easy to imagine a stable social norm – a Nash equilibrium – that achieves this result: women are limited to a certain number of children, and anyone who attempts to violate the norm is put to death along with her children. In practice societies often used methods not so different than this. Marriage was limited and delayed through requirements of substantial accumulation of capital or side-payments as a prerequisite to get married, and unwed mothers were severely punished, in many cases through capital punishment. This seems to be understood by demographic historians such as Bacci (2006), and as well as by Malthus himself – he simply thought that in some long-run evolutionary sense these low population equilibria were unstable. We will argue the opposite. Specifically, our intuition about free resources can be captured by the following conceptual experiment. Imagine a "Malthusian" society with farmers living at the edge of subsistence. Next door live their less numerous but richer neighbors who control their population. What happens when the few but rich neighbors invade the nearly starving farmers? For the farmers to spend time fighting is to take time from farming – that is, to starve. The outcome of this conflict is easy to see.

What this suggests is that what counts for free resources should be aggregate output in excess of subsistence. Specifically if z denotes the total population on a plot of land (which to ease calculations we allow to take on real as well as integer values) we let the function $Y(z)$ be the aggregate output produced on that plot of land and that α is a scalar technology parameter. We denote by B the per capita output needed for subsistence. There is, of course, a big debate about what constitutes subsistence.[11] This debate is primarily an effort to "prove" that prior to the industrial revolution everyone was at subsistence – the more elastic is the notion of subsistence this is easier to "prove" true.[12] Here we have in mind a more traditional notion of subsistence as requirements for survival and reproduction.

In the *Malthusian Game* the N players are families who choose family size, so that $\tilde{A}^i = \{1, 2, \ldots, M\}$ for some M. Families are Malthusian in the sense that their utility comes from having children: $\tilde{u}^i(\tilde{a}_t^{ij}) = \tilde{a}_t^{ij}$. Total population is given by $z_t^j = \sum_{i=1}^N \tilde{a}_t^{ij}$, and free resources (and hence state power) $f(\tilde{a}_t^j) = Y(z_t^j) - Bz_t^j$. We assume that $Y(NM) - BNM > 0$ so that it is not feasible to choose a population so large that it starves.

From our main result we know that stochastic stability requires maximization of state power, which we are assuming is equal to free resources. Consider first a simple technology with two parameters $\alpha > B, \bar{z} > 0$ where $Y(z) = \min\{\alpha z, \alpha\bar{z}\}$. Free resources are $\min\{\alpha z - Bz, \alpha\bar{z} - Bz\}$ which is maximized at $z = \bar{z}$ with corresponding free resources equal to $(\alpha - B)\bar{z}$. Consider technological change increases both $\bar{z}$ and $(\alpha - B)\bar{z}$. Larger $\bar{z}$ means that technological improvement unambiguously increases population, since the free resource maximizing level of population is $\bar{z}$. Free resources go up since they are $(\alpha - B)\bar{z}$, hence the new technology is "improved" in the sense that societies choosing the new technology will supplant those with the old technology. Notice however that per capita output is α and increases in both $\bar{z}$ and $(\alpha - B)\bar{z}$ are consistent with either

[11] Historically, the subsistence level meant "the physical requirements to survive and reproduce." In the hands of modern economic historians such as Clark (2007) "subsistence" has become an elastic concept meaning the "some socially determined level of per capita income above which population decreases and below which it increases." This is somewhat awkward as the cross-sectional evidence is clear that rich countries reproduce as much lower rates than poor ones.

[12] A close reading of the literature reveals serious problems. The most central problem is that at best what is computed is median per capita income – that is, the typical income of a poor person. Of course the upper classes consume considerably more than subsistence so the mean must also be above subsistence. A typical example of this problem is in the classical and much cited Ladurie (1974) study of Languedoc peasants in France. Ignored in this study are the facts that the nobles live above subsistence; that the entire area made substantial payments to the King – and indeed the ability of France to conduct continual wars throughout this period indicates that substantial free resources were available. More serious students of historical per capita GDP such as Maddison (2007) point out the Malthusian bias implicit in conclusions of this type.

α increasing or decreasing. Hence the "improvement" is ambiguous for per capita output and individual welfare. Consider for example the spread of agriculture. Bowles and Choi (2013) argue that this new technology lowered individual welfare even as it enabled greatly increased population density (large $\bar{z}$).[13] Moreover they argue that agriculture spread not because societies saw neighboring societies using a superior technology and adopted it, but rather because societies that adopted agriculture quickly conquered societies that did not. They show how conflict in an evolutionary model with conflict can be used to explain this transition. That simulation result can be understood in simple terms in the light of our theoretical results: if the essence of agricultural technology is to be able to replicate plants hence feed more people, it is characterized by higher $\bar{z}$ hence higher $(\alpha-B)\bar{z}$: it has higher population density and more free resources which it will use to overwhelm hunter gatherer societies. On the other hand, if α is lower in an agrarian society then indeed per-capita income declines.

By contrast we may consider the industrial revolution as raising both $\bar{z}$ and α. In this case population goes up, per capita output and welfare go up – and free resources go up, so the industrial societies overwhelm non-industrial societies militarily – as, with the exception of Japan which voluntarily chose to industrialize – is what happened.

The same story holds with more realistic technologies and a single parameter technological parameter. Suppose that aggregate output $Y(z)=\alpha\tilde{Y}(z)$ where $\tilde{Y}(z)$ is a differentiable, strictly concave and strictly increasing function of population z. We will explore how the (unique since $\tilde{Y}(z)$ is strictly concave) real value of z that maximizes free resources varies with the technological parameter α, and how income per capita goes; since the problem is concave, the optimal integer valued solution must be one of the adjoining grid points.

Recall for comparison the usual Malthusian case where population is so large that income per capita is at subsistence level, that is where the value of z that satisfies $\alpha\tilde{Y}(z)(z)/z=B$. This value is strictly increasing in α and gives the usual Malthusian result: technological change in the long-run leaves per capita income unchanged and leads merely to an increase in population. In the case of maximization of free resources the situation is as follows:

Proposition 3. *The value of z^* that maximizes free resources is strictly increasing in α. Per capita output increases with α if and only if*

$$-\frac{z\tilde{Y}''(z^*)}{\tilde{Y}'(z^*)}>1-\frac{z\tilde{Y}'(z^*)}{\tilde{Y}(z^*)}.$$

Proof. The value z^* is defined implicitly by the equation $\alpha\tilde{Y}'(z)=B$ as a function of α. We want to compute the derivative of this function z and of $\alpha\tilde{Y}(z)/z$ with respect to α. From the condition $\alpha\tilde{Y}'(z)=B$ we get $0=\tilde{Y}'d\alpha+\alpha\tilde{Y}''dz$, whence $dz/d\alpha=-\tilde{Y}'/A\alpha\tilde{Y}''>0$. Computing $d(\alpha\tilde{Y}(z)/z)/d\alpha$ results in the last condition in the statement. □

Corollary 2. *Let again z^* be free resource maximizing, as a function of α. In the Cobb–Douglas case $\tilde{Y}(z)=z^\beta$ per capita output is independent of α. In the logarithmic case $\tilde{Y}(z)=\log(b+z)$, $b>0$ per capita output is increasing for sufficiently large α, while for large enough b it is decreasing for small α and increasing for large α.*

Proof. In the Cobb–Douglas case we have

$$-\frac{z\tilde{Y}''(z)}{\tilde{Y}'(z)}=1-\frac{z\tilde{Y}'(z)}{\tilde{Y}(z)}=1-\beta,$$

so this case is completely neutral, just as in the Malthus case.

In the logarithmic case $\tilde{Y}'(z)=1/(b+z)$, $\tilde{Y}''(z)=-1/(b+z)^2$. The condition in Proposition 3 can be simplified to

$$\frac{z}{b+z}>1-\frac{z}{(b+z)\log(b+z)}$$

which is equivalent to $\log(b+z)<z/b$ because $z\geq1$. Now $\log(b+z)<\log b+z/b$ for all $z>0$, so the above inequality is satisfied for all $z\geq1$ if $b\leq1$. For $b>1$ it is true for z large enough (the RHS goes to ∞ as $z\to\infty$), so it is satisfied for big enough α. Looking at $z=1$ we get

$$b\log(1+b)<1$$

which clearly fails for big enough b. Hence for large b per-capita income first goes down then up. □

Of course there remains the question of whether we should imagine that technology is more like that of Cobb–Douglas or of the logarithmic form in population size. It seems compelling that only so many people can fit on a particular plot of land before production becomes impossible due to overcrowding as in the simple linear technology with a limit. In this case it is easy to see why per capita output must increase with technological improvement: once the upper bound on population is reached there is no point in adding more people regardless of the state of technology. The only way to take advantage of improved technology to get more free resources is through increased per capita output. In other words, we expect that returns to population drop to zero as population grows. While we have not yet reached the unfortunate state of affairs in

[13] Concerning differential effects of technological progress on population size and income per capita, Ashraf and Galor (2011) elaborate evidence of increase in both population and income per capita in the last two thousand years, and estimate that technological progress in this period has had more impact on population size than on income per capita.

which production is impossible due to overcrowding, this argument does indicate some reason to think that returns to population diminish rather quickly as population on a plot of land grows. It suggests that the more rapidly decreasing returns of the logarithmic model may make more sense than the rather slowly decreasing returns of the Cobb–Douglas model.[14]

7. Private information and inefficiency

Free resources are in a sense viewed here as a residual – what is left over after for use in conflict. Subsistence needed to survive and reproduce cannot be diverted to conflict. The same may also be said of incentive payments: if it is necessary to provide incentive payments to get output produced, these payments cannot be diverted to conflict without also losing the output. In general the situation is complex: an example similar to incentive payments is that documented by Weightman (2009) who reports how British workers in the 19th century consumed roast beef. It was a luxury, but it made them stronger, better workers than on the continent where diet was poorer; so presumably it made them better soldiers as well. In general a diet above subsistence may increase free resources because it increases the ability of workers to produce output. However, like incentive payments, the payments that enable this improved diets are not part of free resources.

To get an idea how this works with incentive payments, we retain the setting of a social norm game, but now drop the assumption of perfect observability and examine settings of private information where the folk theorem does not apply. In this section we consider a hidden effort game which is a simple and relatively standard principal-agent type of model of effort provision.

The players in the *hidden effort game* are identical agents. Each agent chooses an effort level $e_t^{ij} \in \{0,1\}$ and as a consequence of this effort either observable output Y or no output is generated. The probability of output Y is π_e. Privately observed effort increases the chance of output, that is $\pi_1 > \pi_0$.

The plot of land as a whole must decide how to tax producers, or equivalently determine the amount $W_t^j \le Y$ paid to each successful agent with the remaining output becoming free resources. This is done by a voting scheme: each agent votes for a wage rate $W_t^{ij} \in \Omega$. Here Ω is a finite set, although as before to simplify computations we will later treat it as continuous – or at least assume that the grid includes the relevant values. We assume that there is a threshold $N > N_3 > N/2$ such that if this many or more agents agree on a wage rate the actual wage rate W_t^j is that rate, while the default if no agreement is reached is $W_t^j = Y$, that is, if there is no agreement on taxes there are no taxes.[15]

We assume that votes cast for participation in a particular society and votes on the payment scheme are perfectly observed. In summary, $\tilde{a}_t^{ij} = (e_t^{ij}, W_t^{ij})$ and $\theta_t^j = (\theta_t^{ij})_{i=1,\dots,N}$ where $\theta_t^{ij} = (W_t^{ij}, o_t^{ij})$ with probability one.

Agents are risk neutral, so their utility is given by $\tilde{u}^i(\tilde{a}_t^{ij}) = -e_t^{ij} + \pi_{e_t^{ij}} W_t^{ij}$. For simplicity we take the subsistence level $B=0$ so that we may ignore the subsistence constraint.

Players can be shunned in the second stage as a consequence of the output they produce in the first stage and it will generally be desirable to do so even on the equilibrium path. Potentially this can interact with the use of shunning as a social sanction for perfectly observed voting behavior since it reduces the amount of "additional" shunning that can be used as punishment. For this reason we now assume that $\tilde{u}^i(\tilde{o}_t^{ij}) = 0$ so that there is no cost or benefit of voting on a particular aggregation, so no need to provide incentives. In the case of voting on tax rates, we note that any tax rate is an equilibrium: if the vote is unanimous by the assumption that the threshold $N^3 < N$ no single agent is decisive, so again no need to provide incentives.

With respect to the cost imposed by shunning we now make the symmetry assumption that $\Pi^i = \Pi$. We continue to assume that shunning has no impact on free resources. Hence the expected value of free resources generated by agent i is $f(\tilde{a}_t^{ij}) = \pi_{e_t^{ij}}(Y - W_t^j)$. Notice that free resources here are essentially profit, so a free resource maximizing society will behave like a profit maximizing principal.

As we have noted any choice of societies and incentive scheme choices are equilibrium choices, so what matters is the incentive constraint that characterizes Nash equilibrium high effort. Let p_t^{ij} and P_t^{ij} denote the shunning probabilities if output is zero or Y. Then the incentive constraint for high effort is

$$(1-\pi_1)[-p_t^{ij}\Pi] + \pi_1[W_t^j - P_t^{ij}\Pi] - 1 \ge (1-\pi_0)[-p_t^{ij}\Pi] + \pi_0[W_t^j - P_t^{ij}\Pi],$$

which setting $\Delta \equiv (\pi_1 - \pi_0)^{-1}$ may be written as

$$[W_t^j + (p_t^{ij} - P_t^{ij})\Pi] \ge \Delta.$$

Our primary result compares the maximization of utility and free resources, with a view to efficiency and shunning.

[14] The mechanism here is not dissimilar to that discussed in Hansen and Prescott (2002): there it is the exhaustion of land that forces a change to a capital based technology that increases per capita income.

[15] This is similar to the approach sometimes taken in the repeated game literature such as Fudenberg et al. (1994). There a mechanism design problem is mapped into a game by adding a stage in which people vote for the preferred mechanism. Either everyone gets some very low level of utility because they disagree, or if they all agree, then the agreed upon mechanism is implemented. In such a game every incentive compatible mechanism is a Nash equilibrium.

Proposition 4. (*i*) *There is a first best* (*utility maximizing*) *Nash equilibrium. In any such equilibrium* $W_t^j = Y$ *so that there are no free resources*; *shunning never occurs and effort is provided if and only* $(\pi_1 - \pi_0)Y \geq 1$, *that is the expected output gain is no smaller than effort cost.* (*ii*) *The free resource maximizing Nash equilibrium requires shunning* (*only*) *agents who provide zero output with probability one. The wage rate is* $W_t^j = \max\{0, \Delta - \Pi\}$ *and effort is provided if and only if either* $\Delta \leq \Pi$ *or* $\Delta > \Pi$ *and* $(\pi_1 - \pi_0)Y \geq \pi_1(\Delta - \Pi)$.

Proof. To maximize expected utility $-e_t^{ij} + \pi_{e_t^{ij}} W_t^j - \Pi[(1-\pi_{e_t^{ij}})p_t^{ij} + \pi_{e_t^{ij}} P_t^{ij}]$ it must clearly be $W^j = Y$ and $p_t^{ij} = P_t^{ij} = 0$, so the problem becomes $\max_{e \in \{0,1\}} -e + \pi_e Y$ which gives $e = 1$ iff $(\pi_1 - \pi_0)Y \geq 1$. Statement (i) follows. Turning to free resources: without effort they are just $\pi_0 Y$. To induce effort one must solve max $\pi_1(Y - W_t^j)$ subject to $W_t^j + (p_t^{ij} - P_t^{ij})\Pi \geq \Delta$; here we want W^j as low as possible so we set $p_t^{ij} = 1, P_t^{ij} = 0$ to ease the incentive constraint, which then becomes $W_t^j \geq \Delta - \Pi$; so to induce effort optimal choice is $W_t^j = \max\{0, \Delta - \Pi\}$, giving the wage rate. From this it follows that if $\Delta \leq \Pi$ we have $W_t^j = 0$ and free resources are $\pi_1 Y > \pi_0 Y$ so that effort is always induced. If on the other hand $\Delta > \Pi$ the wage rate is $W_t^j = \Delta - \Pi$, so free resources are $\pi_1(Y - \Delta + \Pi)$, hence effort is induced iff $\pi_1(Y - \Delta + \Pi) \geq \pi_0 Y$ that is iff $(\pi_1 - \pi_0)Y \geq \pi_1(\Delta - \Pi)$. □

In the case of free resource maximization, if $\Delta \leq \Pi$ then shunning alone is enough to provide incentives. This means effort should be induced even if it is inefficient, that is even if $(\pi_1 - \pi_0)Y < 1$, or equivalently $Y < \Delta$. This can be interpreted as a kind of slavery where punishments (whips) are used to keep people in line. When detection is easy (low Δ) and whips are nasty (Π is large) this gives the most free resources.

The opposite case where output $Y > \Delta$ is large and detection is hard ($\Delta > \Pi$) is characteristic of innovative activity. Notice that $\pi_1 \Delta > 1$. If output is not too large, $Y < \pi_1 \Delta(\Delta - \Pi)$, then free resources are maximized by having no effort provision. The point is that in the innovative environment punishment alone is not enough, we have to provide incentive payments resulting in informational rents to innovators, and these may be sufficiently high, that the increase in output is offset by the increased informational rents and it is better (from a free resource perspective) not to bother.

8. Conclusion

Readers of grand theories of history such as those of McNeil (1963), Cipolla (1965), Diamond (1998) or Acemoglu and Robinson (2012) will not find the idea surprising that ideas are spread by the conquest of the less advanced by the more advanced – indeed it seems almost ubiquitous in their anecdotes and discussions. Missing from these accounts, however, is the notion that it is free resources above and beyond subsistence and incentive payments that matter for the long-term success of societies. In essence, the conclusion of our theory is that evolution favors large expansionary societies made strong by availability of free resources. This is also what historical evidence shows, from old China to the Romans, to modern England and the contemporary United States.

It would be amazing indeed if a simple theory with single scalar variable "free resources" or "state power" could explain all of history. Missing is any account of the geographical barriers that in practice have prevented a single monolithic society from covering the entire globe. Indeed, England, fast behind her water barrier, continually favored the weaker side in continental Europe to prevent a monolithic society from arising there. Perhaps the history of Asia would have been very different if Japan had been geographically capable of playing a similar role in China. Geography may also be intertwined with strategical considerations: the small Kingdom of Sardinia was located between the great powers of Austria and France who could have easily conquered it, but they never did because a small buffer at their borders made attack by one another harder.[16]

Indeed, geography plays a role in a variety of ways: technology matters of course – water barriers matter much less to societies that have boats – and even less if they have airplanes. The libertarian success stories favored by Milton Friedman in Singapore and Hong Kong were also protected – in the case of Hong Kong by the British military, and in the case of Singapore by a water barrier. One aspect of the theory worthy of future exploration is the idea that small geographically protected areas are likely to have a broader range of social arrangements – both efficient and inefficient – being protected from conquest and disruption by neighbors.[17] We examine these issues in greater detail in Levine and Modica (2013).

On the other hand, there are a variety of historical episodes that may be interesting to explore through the lens of free resources. For example, at the beginning of the cold war, technology favored assembly line manufacturing which is relatively amenable to central planning, and so the Soviet Union, a system that excelled at appropriating a high fraction of resources as free, was able to compete successfully with the United States. By contrast as technology changed to favor greater decentralization, it is likely that the enormous growth of GDP in the United States relative to the Soviet Union made it impossible for the Soviet Union to continue to compete, despite its ability to appropriate a very high fraction of total resources. In a similar way, the development of firearms at the end of the medieval period favored moderately skilled mass armies over small highly trained armies of specialists. Hence to generate large free resources, higher per capita income was needed. The ultimate failure of poorly trained peasants to resist moderately trained lower middle class soldiers was seen in

[16] A recent empirical paper on the relation between warfare and institutions in the Italian *Risorgimento*is Dincecco et al. (2011).

[17] The wide range of (admittedly very primitive) social arrangements in New Guinea may be a case in point.

the early 20th century in the defeat of Russia first by Japan, and eventually by Germany which effectively ended the Russian Empire at the battle of Tannenberg.

We should acknowledge also that while conflict is an important force in the spread (and disruption) of institutions and ideas, voluntary movement of the type discussed by Ely (2002) exists as well and provides a force away from free resource maximization and towards efficiency. This suggests a more refined theory in which both free resources and efficiency matter, with the relative strengths of the two depending on the relative importance of ideas spreading through conquest versus voluntary movement.

In summary, the notion of evolution through contacts and conflicts between societies leads to a simple and in our view plausible model of stochastically stable states that maximize free resources. Implications of the theory range from determining the level of population to the type of technologies and institutions we may expect to find.

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Appendix A

Proposition (*Proposition 1 in text*). *$\sigma \in S[0,J]$ if and only if: (i) σ is a singleton, that is, $\sigma = \{s_t\}$, (ii) s_t is a Nash state, and (iii) s_t has either no expansionary society, or a single expansionary society such that all other societies (if any) have positive resistance to disruption.*

Proof. First we observe that the s_t as described are absorbing states of the Markov chain, hence certainly in $S[0,J]$. This is trivial, since by assumption no disruption is possible at these states. To prove the theorem it is sufficient to show that from any other initial condition there is a positive probability of reaching one of these absorbing states. This rules out existence of other ergodic classes.

We show that one of these absorbing states has positive probability of being reached from any initial condition. First notice that there is a positive probability that for $T+L+1$ periods no plot is disrupted. During such a period a quiet plot remains quiet. In a plot j in which some player is not quiet there is a positive probability that all players on that plot will not be quiet the following period. There is then a positive probability that for the next $T+L$ periods all players will play a steady state Nash equilibrium profile and the plot will become quiet. Since this is true of all plots and there are finitely many of them, there is a positive probability that after $T+L+1$ periods the state will be a Nash state.

Suppose we begin in a Nash state which is not one of the described absorbing states. Then there is some expansionary society x that has the most free resources among all expansionary societies that are present in that state (there may be more than one such). If there is more than one expansionary society, one of them has free resources relative to some other of at least 1, and hence by Assumption 3 it has positive probability of becoming the sole expansionary society. Hence multiple expansionary societies are transitory.

If there is no other expansionary society, by assumption one of them has positive probability of being disrupted. Subsequently the disrupted plot has positive probability of joining society x and so there is positive probability of moving to a steady state Nash equilibrium where x has one more plot. By the second part of Assumption 3 we can repeat the process (still with positive probability) until the absorbing state in which $J(x) = J$ is reached. □

To prove the main theorem we will now apply a method of Friedlin and Wentzell (1984) described in Young (1993) to analyze the case $\mathcal{E} > 0$ and the limit as $\mathcal{E} \to 0$. We use the characterization of stochastically stable states given by Young (1993). Let $\mathcal{T}$ be a tree whose nodes are the set $S[0,J]$ with any set of edges. We denote by $D(s)$ the unique node from s in the direction of the root. An s-tree is a tree whose root is s, denoted $\mathcal{T}(s)$. For any two points $s_0, s_t \in S[0,J]$ we define the resistance as follows. First, a path from s_0 to s_t is a sequence of points $s_0, \ldots, s_t \in S$, where the transition from s_τ to $s_{\tau+1}$ has positive probability for $\mathcal{E} > 0$. The resistance of the path is the sum of resistances between points in the path $\sum_{\tau=0}^{t-1} r(s_\tau, s_{\tau+1})$. The resistance $r(s_0, s_t)$ is the least resistance of any path from s_0 to s_t. The resistance $r(\mathcal{T}(s_t))$ of the s_t-tree $\mathcal{T}(s_t)$ is the sum over non-root nodes s_τ of $r(s_\tau, D(s_\tau))$: $r(\mathcal{T}(s_t)) = \sum_{s_\tau \in S[0,J] \setminus s_t} r(s_\tau, D(s_\tau))$. Finally, the resistance $r(s_t)$ is the least resistance of all the s_t-trees. The following Theorem is proved in Young (1993).

Theorem 4. *s_t is a stochastically stable state if and only if $s_t \in S[0,J]$ and $r(s_t) = \min_{s_\tau \in S[0,J]} r(s_\tau)$.*

We can provide a lower bound on the resistance of the trees on $S[0,J]$. For any state $s_t \in S[0,J]$ and any subset $\tilde{S} \subset S[0,J]$ define the least resistance states $LR(s_t, \tilde{S}) \subseteq \tilde{S}$ to be the collection of states $s_\tau \in S[0,J] \setminus s_t$ such that $r(s_t, s_\tau) \le r(s_t, \tilde{s}_\tau)$ for all $\tilde{s}_\tau \in \tilde{S}$.

Let $lr(s_t, \tilde{S})$ be the corresponding least resistance. When $\tilde{S} = S[0,J]$ this is equivalent to Ellison (2000)'s notion of the radius of a state.

For each expansionary society x for which the corresponding set of monolithic states $S(x)$ is nonempty pick an $s_t \in S(x)$ and define $l(s_t) \equiv \min_{\tilde{s}_t \in S(x)} lr(\tilde{s}_t, S[0,J] \backslash S(x))$ – the minimum least resistance to pass from a state in $S(x)$ to one *outside* of it; for the remaining states (if any) $s_t \in S(x)$ define $l(s_t) \equiv \min_{\tilde{s}_t \in S(x)} lr(\tilde{s}_t, S[0,J])$ – the minimum least resistance to go from a state in $S(x)$ to any state in $S[0,J]$, which in fact is always 1. For states s_t which belong to no $S(x)$ define $l(s_t) \equiv lr(s_t, S[0,J])$. Define $ml \equiv \sum_{s_\tau \in S[0,J]} l(s_\tau) - \max_{s_\tau \in S[0,J]} l(s_\tau)$, that is the sum of all the $l(s_\tau)$'s except the highest.

Lemma 1. *Any s_t-tree $\mathcal{T}(s_t)$ satisfies $r(\mathcal{T}(s_t)) \geq ml$.*

For the sake of clarity we observe that if no state belonged to an $S(x)$ the lemma would be trivial, because $r(\mathcal{T}(s_t)) = \sum_{s_\tau \in S[0,J] \backslash s_t} r(s_\tau, D(s_\tau))$ and by construction $r(s_\tau, D(s_\tau)) \geq \min_{s \in S[0,J]} r(s_\tau, s) = lr(s_t, S[0,J]) = l(s_\tau)$ so $r(\mathcal{T}(s_t)) \geq \sum_{s_\tau \in S[0,J] \backslash s_t} l(s_\tau)$, which is the sum of all the $l(s_\tau)$'s except $l(s_t)$; so either $l(s_t)$ is highest and then the sum is ml by definition, or $l(s_t)$ is not highest and then the sum is larger than ml (because it has the highest term in and leaves out a smaller one). A slight complication arises because for the selected element s_t in the non-empty $S(x)$ we have defined $l(s_t)$ to be a number possibly larger than $\min_{s \in S[0,J]} r(s_t, s)$ (for reasons which will be clear in the proof of the main result). To this we turn in the proof of the lemma.

Proof (*Proof of Lemma*). For any s_t-tree $\mathcal{T}(s_t)$ we have $r(\mathcal{T}(s_t)) = \sum_{s_\tau \in S[0,J] \backslash s_t} r(s_\tau, D(s_\tau))$. By construction it is $r(s_\tau, D(s_\tau)) \geq l(s_\tau)$ for all s_τ except possibly for the designated states in the non-empty $S(x)$'s in case the transition $(s_\tau, D(s_\tau))$ on the tree takes place inside their $S(x)$. Consider then such a designated state s^*. If its $S(x)$ contains also the root s_t then it must be $r(s^*, D(s^*)) \geq l(s_t)$ and then we get $r(\mathcal{T}(s_t)) \geq \sum_{s_\tau \in S[0,J] \backslash s^*} l(s_\tau)$ which is no smaller than ml because the latter leaves out the highest term. If on the other hand the $S(x)$ containing s^* does not contain the root s_t then for some $s_\tau \in S(x)$ the transition $(s_\tau, D(s_\tau))$ must take s_τ out of $S(x)$; then from $r(s_\tau, D(s_\tau)) \geq l(s^*)$ and $r(s^*, D(s^*)) \geq l(s_\tau)$ we get $r(s_\tau, D(s_\tau)) + r(s^*, D(s^*)) \geq l(s_\tau) + l(s^*)$, which again implies the inequality in the statement. □

The bound established in the Lemma is not generally a useful one, but in the current setting we shall show that there is a tree that achieves this bound. Such a tree is necessarily a least resistance tree.

The central theorem of the paper, Theorem 3 in text, itself has two parts, which we cover in the next result and the corollary which follows. We recall that a stochastically stable society is one for which all the corresponding monolithic states $S(x)$ are stochastically stable; and that a strongest expansionary society is one whose minimum free resources among the admitted profiles (weakest link of chain) is highest.

Theorem 5. *If x is a strongest expansionary society then it is stochastically stable.*

Proof. Recall that in the least (average per plot) state power states in a society, in particular in a strongest expansionary society x, the profiles played in all plots must have the same level of state power. In some such states the profiles may be different from one another in the society's plots, but in others all plots play the same least state power profile. Such states we consider first: we show that for some such $s_t(x) \in S(x)$ it is possible to build a tree $\mathcal{T}(s_t(x))$ such that $r(\mathcal{T}(s_t(x))) = ml$. This achieves the lower bound by Lemma 1, so must be a least resistance tree. Then we show how to rearrange this tree without increasing the cost so that any state $s_t \in S(x)$ is the root.

To build $\mathcal{T}(s_t(x))$ we show how to connect every node $s_\tau \in S[0,J] \backslash s_t(x)$ into the tree. The aim is to obtain

$$\sum_{s_\tau \in S[0,J] \backslash s_t(x)} r(s_\tau, D(s_\tau)) = \sum_{s_\tau \in S[0,J]} l(s_\tau) - \max_{s_\tau \in S[0,J]} l(s_\tau).$$

We start with the non-expansionary states. For such an s_τ the least cost $l(s_\tau)$ is established by a single plot of land being disrupted to any Nash state for that plot: this has a resistance of 1, the least possible resistance of any transition from a non-expansionary state. In particular, we may connect the non-expansionary state to the state in which the lowest numbered plot of land is disrupted to an expansionary profile X. Hence, since X is by construction expansionary, each non-expansionary state is connected at least cost to a mixed state.

For mixed states, a least cost transition is for a non-expansionary society to have a single plot disrupted – to any particular target. The transition has resistance no greater than 1. Among the plots in non-expansionary societies that have the greatest chance of disruption suppose that the lowest numbered plot is disrupted to the profile of the highest numbered plot of the expansionary society. Notice that this process cannot result in a cycle, since the number of plots owned by the expansionary society increases by one at each step. In this way, each mixed state is connected by a sequence of least cost transitions to a state in a monolithic expansionary state.

This reduces the problem to sub-trees on the states corresponding to the expansionary societies y.

For each expansionary society y fix a least resistance profile $Y \in \chi^{-1}(y)$. First we deal with $s_t \in S(y)$ where not all plots are playing the given least resistance profile Y. For these states disrupting the lowest numbered plot of land in which the state is not Y to the state Y has a cost of 1 which is the least possible, since societies to not have conflict within themselves.

Finally we consider, for any y, the unique state $s_t(y) \in S(y)$ where all plots play Y. We will show that there is a path from $s_t(y)$ to any $s_t(z)$ where $z \neq y$ that achieves $\min_{\tilde{s}_t \in S(y)} lr(\tilde{s}_t, S[0,J] \backslash S(y))$. We designate $s_t(x)$ as the root of our proposed tree, and for $y \neq x$ we take $z{=}x$, that is these other minimal state power monolithic expansionary states should go directly to the root.

It is convenient to illustrate the least cost path from $s_t(y)$ to $s_t(z)$ by means of a diagram. We may write $s_t(y) = YYYY\ldots Y$. Let A be a profile on a single plot that maximizes state power among all possible profiles, incentive compatible or not. We refer to this as the *barbarian horde*. Then the transitions are

$$
\begin{aligned}
&YYYY\ldots Y\\
&AYYY\ldots Y\\
&ZZYY\ldots Y\\
&AAYY\ldots Y\\
&ZZZY\ldots Y\\
&\vdots\\
&ZZZZ\ldots Z\\
&\vdots\\
&ZZZZ\ldots Z
\end{aligned}
$$

Notice that when a Y is replaced by a Z the most possible state power of an opponent A faces the least possible state power of any society of the given size in $S(y)$. Notice second that going down a column, until the final stage we alternate $AZAZ\ldots$. Since $L \geq 2$ this assures that the plot is never quiet, and so all transitions on that plot have 0 resistance: the only costly resistance is when a plot Y is converted to Z, which has resistance 1. In the final stage, we hold what happens fixed with all plots playing Z which is by assumption a Nash equilibrium, and the transitions have no cost, so there should be L final transitions after which all the plots become quiet and the state $s_t(z)$ is achieved. Thus we have shown that for any $S(y)$ we have $\sum_{s_t \in S(y)} r(s_\tau, D(s_\tau)) = \sum_{s_t \in S(y)} l(s_\tau)$ on the tree.

The construction also shows that $s_t(x)$ achieves $\max_{s_\tau \in S[0,J]} l(s_\tau)$ since it is the root which is discarded; hence for the tree just described we have $r(\mathcal{T}(s_t)) = ml$ which is what was to be shown.

Finally, we show how to rearrange the tree so that any $s_t \in S(x)$ is at the root, without increasing resistance. This is relatively trivial, since if we put s_t at the root, and connect the remaining $s_\tau \in S(x)$ to the root sequentially by replacing the profile on each plot by the corresponding element of s_t the cost of all these transitions is 1 exactly as in the tree $\mathcal{T}(s_t(x))$. □

Corollary 3. *For J large enough every stochastically stable state $s_t \in S(x)$ for some strongest expansionary society x.*

Proof. It follows from the proof of Theorem 5 that if s_t is the root of a least resistance tree, the lower bound must be achieved, and this is possible only if $l(s_t) = \max_{s_t \in S[0,J]} l(s_t)$. We first show that for large J monolithic states have $l(s_t) > 1$, hence (since the other states have $l(s_t) \leq 1$) only they can be stochastically stable. The claim $l(s_t) > 1$ amounts to asserting that for large J resistance of a monolithic state x to disruption by a barbarian horde x' is positive for at least the first two plots; but as J grows large $2/J \to 0$, so (using the last limit in Assumption 4) the ratio of state power $\phi = F(x', a_t, \omega_t)/F(x, a_t, \omega_t) \to 0$, and for small ϕ resistance is positive by assumption.

Finally observe that the possibility that a society x with $f(x) < \max_{x' > 0} f(x')$ can have $l(s_t) = \max_{s_t \in S[0,J]} l(s_t)$ only because of the round-off error caused by the discrete size of the plots (which makes the barbarian horde jump above the threshold $\overline{\phi}$ in a certain number of steps); but as J grows large this error goes to zero because each conquered plot makes ϕ move less. From this the result follows.

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