Notes on Correlated Equilibrium[1]

1 The notion and first examples and results

We start with a normal form game $G = \langle N, (A_i), (u_i)\rangle$. We denote as usual by $A = \times_i A_i$ the set of action profiles a, and write $a = (a_i, a_{-i})$ when we want to single out the action of a player i.

The idea of correlated equilibrium is that players use a randomization device to coordinate their actions. The device is meant to be operated by an outside "randomizer". The randomizer selects a probability distribution π on A, draws from π, observes the realized a, and reveals a_i and only a_i to player i, for each i. This is interpreted as a recommendation; i can follow it and play a_i or deviate and take another action. Equilibrium is defined (as usual) by the condition that no player wants to deviate from the recommendation if the others are complying. Let $\pi(a_i) = \sum_{a_{-i}\in A_{-i}} \pi(a_i, a_{-i})$ denote the probability of recommendation a_i. If $\pi(a_i) > 0$ we can define the conditional probability of a_{-i} given a_i: $\pi(a_{-i} \mid a_i) = \pi(a_i, a_{-i})/\pi(a_i)$. We adopt the following

Definition. A correlated equilibrium of G is a probability π on A such that for any i and a_i with $\pi(a_i) > 0$

$$\sum_{a_{-i}\in A_{-i}} \pi(a_{-i} \mid a_i) u_i(a_i, a_{-i}) \geq \sum_{a_{-i}\in A_{-i}} \pi(a_{-i} \mid a_i) u_i(a_i', a_{-i}) \quad \forall a_i' \in A_i. \tag{1}$$

Given recommendation a_i and assuming all $j \neq i$ are following the recommended actions, the left member is the what player i gets if she complies too (in expected terms of course); the right member is what she gets if she deviates to a_i'. Note that the conditional probability is the same in both sides. Observe also that also Nash equilibrium can be viewed as a probability on A satisfying the no-deviation property - but with the restriction $\pi(a) = \Pi_{i=1}^n \pi(a_i)$, which is not imposed here.

Example (Battle of Sexes)**.** Consider the battle of sexes on the left (player 1 is the boy who prefers football to ballet) and the π on the right:

	F	B
f	$2,1$	$0,0$
b	$0,0$	$1,2$

	F	B
f	$1/2$	0
b	0	$1/2$

All $\pi(a_i)$ are positive for both players, for example for 1 it is $\pi(f) = \pi(fF) + \pi(fB) = 1/2 = \pi(b)$. Still considering player 1 we have $\pi(F \mid f) = \pi(B \mid b) = 1$. Suppose $a = fF$; he is

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[1] S Modica 2024. As usual the exposition is based on Osborne-Rubinstein *A Course on Game Theory.*

recommended to play f; if he plays f she gets 2, while deviating to b he gets 0 (because the girl is playing F for sure); analogously he will not want to deviate if instructed to play b. The same argument holds for player 2 (write it down!). So the displayed π is an equilibrium.

Example (Another chicken game)**.** Consider the game on the left and the π on the right:

	L	R
U	$5,1$	$0,0$
D	$4,4$	$1,5$

	L	R
U	$1/3$	0
D	$1/3$	$1/3$

All $\pi(a_i)$ are positive for both players. For player 1 $\pi(L \mid U) = 1$, and $\pi(L \mid D) = \pi(R \mid D) = 1/2$; she clearly does not want to deviate if she is instructed to play U; if she is recommended to play D the no-deviation condition is $\frac{1}{2} * 4 + \frac{1}{2} * 1 \geq \frac{1}{2} * 5 + \frac{1}{2} * 0$, which is true. Analogously, player 2 does not have profitable deviations whatever she is recommended to play. Thus the displayed π is a correlated equilibrium. The expected payoff for each player in equilibrium is $[5 + 4 + 1]/3 = 10/3$.

As an exercise please check that another correlated equilibrium (yielding each player expected payoff 3) is $\pi(UL) = \pi(DR) = 1/2$. But we should not think that any π is a correlated equilibrium. Let us investigate the distributions of the form

	L	R
U	$\frac{1-p}{2}$	0
D	p	$\frac{1-p}{2}$

as p varies. We have see the cases $p = 0$ and $p = 1/3$. We would like to have p as high as possible because DL gives the highest average payoff to the players. But for example $p = 1$ won't do: if $\pi(DL) = 1$ then if player 1 is told to play D she will deduce that 2 (complying to the recommendation) is playing L, hence she would deviate to U. We look for an upper bound for p.

With the π as in the table $\pi(U) = \frac{1-p}{2}$ and $\pi(D) = p + \frac{1-p}{2} = \frac{1+p}{2}$. If player 1 is told U, since $\pi(L \mid U) = 1$ she is sure 2 (complying) plays L; so complying gives 5, better than not (which gives 4). If she is told D then $\pi(L \mid D) = p/\pi(D)$ and $\pi(R \mid D) = \frac{1-p}{2}/\pi(D)$. Following the recommendation yields $4 * p/\pi(D) + 1 * \frac{1-p}{2}/\pi(D)$ while deviating gives $5 * p/\pi(D) + 0 * \frac{1-p}{2}/\pi(D)$; therefore the no-deviation condition is $4p + \frac{1-p}{2} \geq 5p$ that is $0 \leq p \leq 1/3$. It is the whole interval between the two extremes we had seen before. These are the equilibria, since player 2 has the same restrictions as 1. No $p > 1/3$ is equilibrium. The expected payoff of player 1 in these equilibria (same as 2) is $4p + 6\frac{1-p}{2} = 3 + p$, increasing in p; therefore its minimum is 3 and the maximum 10/3. In particular the uniform distribution we had seen gave actually the highest achievable payoff compatible with individual incentives.

The set of correlated equilibria contains the Nash profiles

The following result formalizes the fact that correlated equilibrium is a weakening of Nash, in the sense that the no-deviation restrictions are weaker. So all Nash profiles satisfy them, and there are non-Nash profiles which satisfies them too.

Proposition. *Take a Nash equilibrium* α *and let* $\pi(a) = \alpha(a)$. *Then* π *is a correlated equilibrium.*

Proof. In this case $\pi(a) = \Pi_i \alpha_i(a_i)$, so $\pi(a_i) = \alpha_i(a_i)$ (apply the definition). This implies that any a_i with $\pi(a_i) > 0$ is a best response to α_{-i}. Also $\pi(a_{-i} \mid a_i) = \Pi_{j \neq i} \alpha_j(a_j)$, and therefore $\sum_{a_{-i} \in A_{-i}} \pi(a_{-i} \mid a_i) u_i(a_i, a_{-i}) = U_i(a_i, \alpha_{-i})$. Since a_i is best response (1) is satisfied. □

Technical note

The expectation in (1) is taken after the recommendation has arrived. Note however that the expected payoff 10/3 above is computed ex ante. In this note we check that the following ex ante definition is equivalent to the one we adopt. Of course before a_i is known a strategy of player i is a map $d_i \colon A_i \to A_i$ specifying the action $d(a_i)$ she will take if the recommendation is a_i. Here deviating at a_i means $d(a_i) \neq a_i$.

Alternative Definition. A correlated equilibrium of G is a probability π on A such that for any i, for any map $d_i \colon A_i \to A_i$

$$\sum_{a \in A} \pi(a) u_i(a_i, a_{-i}) \geq \sum_{a \in A} \pi(a) u_i(d_i(a_i), a_{-i}). \tag{2}$$

This says that π is a correlated equilibrium if the strategy $d_i(a_i) = a_i \ \forall a_i$ is optimal for each player. To see that (2) is equivalent to (1) observe that (2) may be rewritten as

$$\sum_{a_i \in A_i} \sum_{a_{-i} \in A_{-i}} \pi(a_i, a_{-i}) u_i(a_i, a_{-i}) \geq \sum_{a_i \in A_i} \sum_{a_{-i} \in A_{-i}} \pi(a_i, a_{-i}) u_i(d_i(a_i), a_{-i}). \tag{3}$$

Now if a_i is such that $\pi(a_i) = 0$ then the inner sums on both sides are zero; so the relevant a_i's are only those for which $\pi(a_i) > 0$; therefore we may rewrite (3) as

$$\sum_{a_i \in A_i} \pi(a_i) \sum_{a_{-i} \in A_{-i}} \pi(a_{-i} \mid a_i) u_i(a_i, a_{-i}) \geq \sum_{a_i \in A_i} \pi(a_i) \sum_{a_{-i} \in A_{-i}} \pi(a_{-i} \mid a_i) u_i(d_i(a_i), a_{-i}) \tag{4}$$

for all a_i *such that* $\pi(a_i) > 0$. Condition (4) is equivalent to (1): if (1) holds then $d_i(a_i) = a_i$ is optimal for all i so (4) holds too; on the other hand if there is and a_i and a_i' for which (1) fails then (4) is violated by $d_i(a_i) = a_i'$.

2 A few more examples

An example by Moulin and Vial

This is a two-player game where each has three actions, with the payoff matrix on the left below:

	L	C	R
T	$0, 0$	$4, 2$	$2, 4$
M	$2, 4$	$0, 0$	$4, 2$
B	$4, 2$	$2, 4$	$0, 0$

	L	C	R
T	0	1/6	1/6
M	1/6	0	1/6
B	1/6	1/6	0

From the symmetry of the game we deduce that the Nash equilibrium must be symmetric. Letting $(p, q, 1 - p - q)$ denote 2's mixture you should check that Nash is then $p = q = 1/3$. Thus the Nash payoff is 2 for each player. We claim that the distribution on the right table above is a correlated equilibrium.

To show this we must check the no-deviation property. Let us look at player 1 for example; when told to play T she knows that 2 is playing C or R with equal probability; so playing as instructed yields her 3; if she plays M she gets $1/2 * 0 + 1/2 * 4 = 2$, and if she plays B she gets $1/2 * 2 + 1/2 * 0 = 1$; so she would not want to deviate. The other checks are analogous. This equilibrium yields expected payoff 3 for each player, which is 50% higher than the Nash payoff. Seen otherwise, each player would pay up to 1 (utility unit) to an external randomizer.

Another chicken

We put different numbers for a change. Actions Stop or Go, payoff matrix

	G	S
G	$-1, -1$	$1.5, 0$
S	$0, 1.5$	$1, 1$

We know the Nash equilibria of this game: the two asymmetric off-diagonal outcomes, plus the symmetric mixed equilibrium which is easily computed to be $p = q = 1/3$. Expected payoff in this equilibrium is the same for both players, and as you can check it is 2/3. In the two pure strategy equilibria the average payoff of the players is $1/2 * 3/2 + 1/2 * 0 = 3/4$.

Let us see what we can do with correlated equilibrium. We want to avoid the GG profile, so we set up a random device with probabilities $\{p, q, r\}$ as in the table below.

	G	S
G	0	r
S	p	q

We have to determine values p, q (being $r = 1-p-q$) to get a correlated equilibrium. We look for an equilibrium where both players get the same payoff. Therefore we must have

$$1.5r + q = 1.5p + q.$$

This implies $p = r$,[2] which since $r = 1-p-q$ in turn implies $q = 1-2p$; from this we deduce that the value of the above payoffs is

$$1.5p + q = 1.5p + 1 - 2p = 1 - p/2.$$

So to maximize the common equilibrium payoff p must be as small as possible, which since $q = 1-2p$ also says q as large as possible.

Let us then see what the no-deviation property entails. For player 1, if he is told to play G he should comply since that is as good as it can get; if he is told S then conditional probabilities are $p/(p+q), q/(p+q)$ and he should (weakly) prefer S to G assuming 2 follows recommendation; therefore it must be

$$1 \cdot \frac{q}{p+q} \geq (-1) \cdot \frac{p}{p+q} + \frac{3}{2} \cdot \frac{q}{p+q} \quad \text{that is} \quad q \leq 2p.$$

From $q = 1-2p$ we get $1-2p \leq 2p$ or $p \geq 1/4$. We conclude that the highest common payoff compatible with player 1 not having a profitable deviation is reached for $p = 1/4 = r$ and $q = 1/2$.

For player 2 analogously if she is instructed to play S she should prefer S to G that is (omitting the division by 2)

$$q \geq (-1) \cdot r + \frac{3}{2} \cdot q \quad \text{that is} \quad q \leq 2r = 2p.$$

Thus at $p = 1/4$ and $q = 1/2$ player 2 does not want to deviate. We conclude that the highest common expected payoff in correlated equilibrium is reached with the distribution $p = r = 1/4$ and $q = 1/2$.

We found above that this common payoff is $1-p/2$; with $p = 1/4$ this is $7/8 = 21/24$. The highest Nash payoff was $3/4 = 18/24$. So the correlated equilibrium payoff is approximately 16.6% higher. It is almost 30% higher than in the symmetric mixed Nash, which was $2/3 = 16/24$.

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[2] Note that we are finding that what we assumed in the previous chicken example - that both off-diagonal profiles had the same probability - *must* hold in chicken if we want an equilibrium where both players get the same payoff.

The Prisoners Dilemma and dominated strategies

In a correlated equilibrium π a dominated action a_i cannot be played with positive probability, where dominated means that there is an a_i' such that $u_i(a_i', a_{-i}) > u_i(a_i, a_{-i})$ for all a_{-i}. Indeed if $\pi(a_i) > 0$ and a_i is dominated, by playing a_i you get $\sum_{a_{-i} \in A_{-i}} \pi(a_{-i} \mid a_i) u_i(a_i, a_{-i})$ and some of the $\pi(a_{-i} \mid a_i)$ must be positive because $\sum_{a_{-i} \in A_{-i}} \pi(a_{-i} \mid a_i) = 1$; this implies that (1) fails.

This result implies in particular that in the prisoners dilemma there are no correlated equilibria other than the Nash equilibrium.

More information may hurt: OR exercise 48.1

This is a three-player game where player 1 chooses rows, 2 columns and 3 one of the three matrices (A, B, C), with payoffs as shown below:

	L	R
U	$0,0,3$	$0,0,0$
D	$1,0,0$	$0,0,0$

A

	L	R
U	$2,2,2$	$0,0,0$
D	$0,0,0$	$2,2,2$

B

	L	R
U	$0,0,0$	$0,0,0$
D	$0,1,0$	$0,0,3$

C

First let us look at the pure Nash equilibria. In the usual way we find

	L	R
U	$0,0^*,3^*$	$0^*,0^*,0^*$
D	$1^*,0^*,0^*$	$0^*,0^*,0$

	L	R
U	$2^*,2^*,2$	$0,0,0^*$
D	$0,0,0^*$	$2^*,2^*,2$

	L	R
U	$0^*,0^*,0$	$0^*,0^*,0^*$
D	$0^*,1^*,0^*$	$0^*,0,3^*$

so the equilibria are URA etc. The equilibrium payoffs are $(0,0,0), (1,0,0), (0,1,0)$. Player 3 always gets zero, and the payoff $(2,2,2)$ which is higher for all is not an equilibrium.[3]

The distribution π given by $\pi(ULB) = \pi(DRB) = 1/2$ is a correlated equilibrium: Player 3 is told B and assesses equal conditional probabilities for UL and DR, so by deviating she gets $1.5 < 2$; Player 1 and 2 deduce the realized profile with certainty whatever they are told and they get zero if they deviate.

The interesting thing here is that player 3 obtains a high payoff *because she does not know* what the others are playing. If she did we would be back to Nash and she would get zero. So in interactive contexts *more information may hurt.*

This phenomenon can never happen in decision problems involving a single agent: in that case if you have some information on the realized state you can condition your choice on that information, and you may always ignore it and choose as if you did not get that information. A formalization of this argument is given in footnote.[4]

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[3] We should check payoffs in the mixed equilibria. We'll do it in the next example.

[4] Given are: a state space S and a set of actions $F = \{f \mid f\colon S \to \mathbb{R}\}$, and you choose a preferred $f^* \in F$. Partial information means that you get to know which event E_i of a partition $\{E_1, \dots E_n\}$ of S has occurred. Now can still choose f^* ignoring information, but you can also choose f_i if $s \in E_i$ for $i = 1, \dots, n$, with

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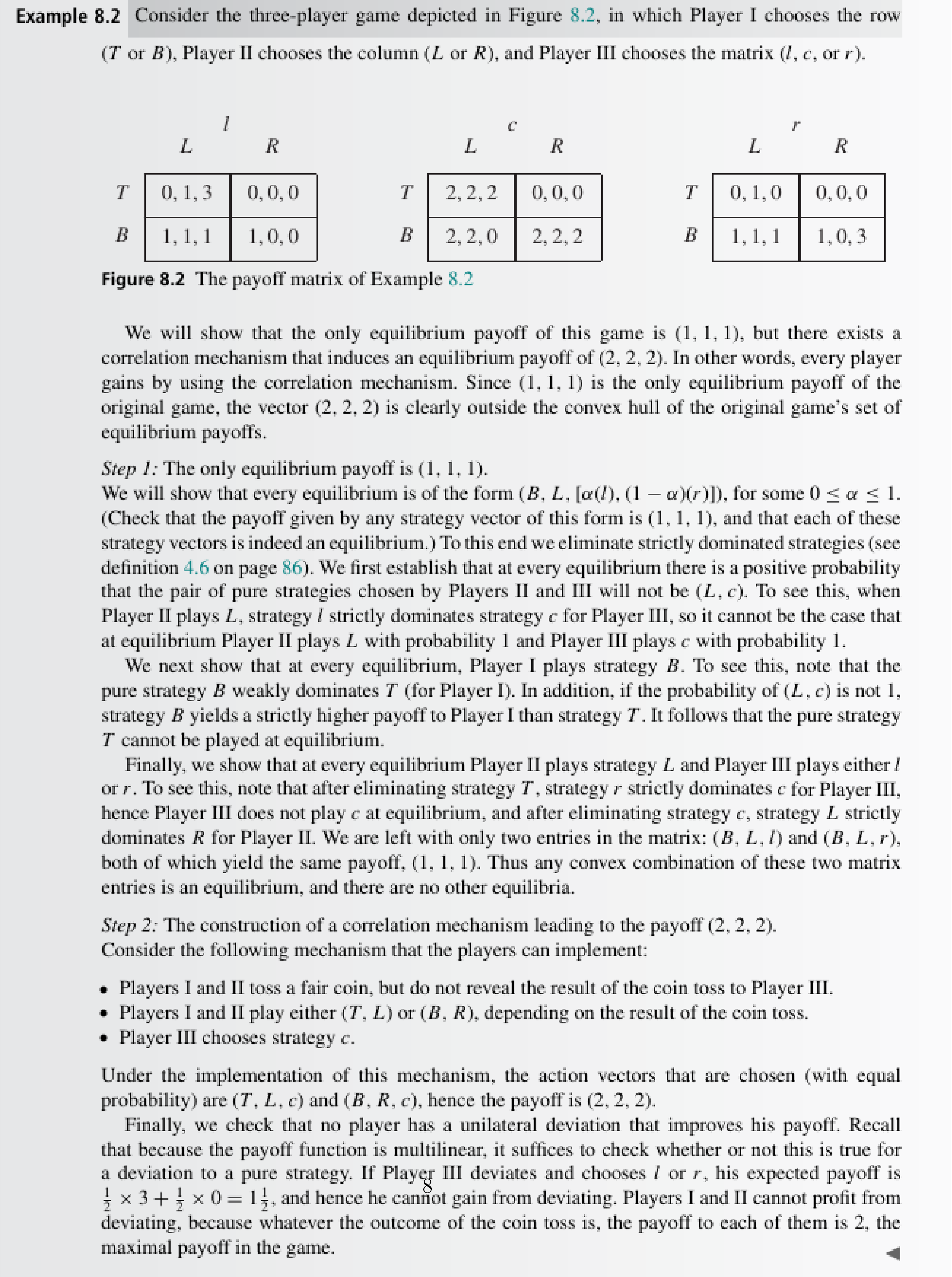
possibly $f_i \neq f_j$ for some i, j. The set of possible choices simply becomes larger, so the realized payoff cannot be lower.

A similar example from Maschler-Solan-Zamir, *Game Theory*, CUP 2013



Example 8.2 Consider the three-player game depicted in Figure 8.2, in which Player I chooses the row (T or B), Player II chooses the column (L or R), and Player III chooses the matrix (l, c, or r).

$$\begin{array}{c|c|c|}
\multicolumn{3}{c}{l} \\
 & L & R \\ \hline
T & 0,1,3 & 0,0,0 \\ \hline
B & 1,1,1 & 1,0,0 \\ \hline
\end{array}
\qquad
\begin{array}{c|c|c|}
\multicolumn{3}{c}{c} \\
 & L & R \\ \hline
T & 2,2,2 & 0,0,0 \\ \hline
B & 2,2,0 & 2,2,2 \\ \hline
\end{array}
\qquad
\begin{array}{c|c|c|}
\multicolumn{3}{c}{r} \\
 & L & R \\ \hline
T & 0,1,0 & 0,0,0 \\ \hline
B & 1,1,1 & 1,0,3 \\ \hline
\end{array}$$

Figure 8.2 The payoff matrix of Example 8.2

We will show that the only equilibrium payoff of this game is $(1, 1, 1)$, but there exists a correlation mechanism that induces an equilibrium payoff of $(2, 2, 2)$. In other words, every player gains by using the correlation mechanism. Since $(1, 1, 1)$ is the only equilibrium payoff of the original game, the vector $(2, 2, 2)$ is clearly outside the convex hull of the original game's set of equilibrium payoffs.

Step 1: The only equilibrium payoff is $(1, 1, 1)$.
We will show that every equilibrium is of the form $(B, L, [\alpha(l), (1-\alpha)(r)])$, for some $0 \leq \alpha \leq 1$. (Check that the payoff given by any strategy vector of this form is $(1, 1, 1)$, and that each of these strategy vectors is indeed an equilibrium.) To this end we eliminate strictly dominated strategies (see definition 4.6 on page 86). We first establish that at every equilibrium there is a positive probability that the pair of pure strategies chosen by Players II and III will not be (L, c). To see this, when Player II plays L, strategy l strictly dominates strategy c for Player III, so it cannot be the case that at equilibrium Player II plays L with probability 1 and Player III plays c with probability 1.

We next show that at every equilibrium, Player I plays strategy B. To see this, note that the pure strategy B weakly dominates T (for Player I). In addition, if the probability of (L, c) is not 1, strategy B yields a strictly higher payoff to Player I than strategy T. It follows that the pure strategy T cannot be played at equilibrium.

Finally, we show that at every equilibrium Player II plays strategy L and Player III plays either l or r. To see this, note that after eliminating strategy T, strategy r strictly dominates c for Player III, hence Player III does not play c at equilibrium, and after eliminating strategy c, strategy L strictly dominates R for Player II. We are left with only two entries in the matrix: (B, L, l) and (B, L, r), both of which yield the same payoff, $(1, 1, 1)$. Thus any convex combination of these two matrix entries is an equilibrium, and there are no other equilibria.

Step 2: The construction of a correlation mechanism leading to the payoff $(2, 2, 2)$.
Consider the following mechanism that the players can implement:

- Players I and II toss a fair coin, but do not reveal the result of the coin toss to Player III.
- Players I and II play either (T, L) or (B, R), depending on the result of the coin toss.
- Player III chooses strategy c.

Under the implementation of this mechanism, the action vectors that are chosen (with equal probability) are (T, L, c) and (B, R, c), hence the payoff is $(2, 2, 2)$.

Finally, we check that no player has a unilateral deviation that improves his payoff. Recall that because the payoff function is multilinear, it suffices to check whether or not this is true for a deviation to a pure strategy. If Player III deviates and chooses l or r, his expected payoff is $\frac{1}{2} \times 3 + \frac{1}{2} \times 0 = 1\frac{1}{2}$, and hence he cannot gain from deviating. Players I and II cannot profit from deviating, because whatever the outcome of the coin toss is, the payoff to each of them is 2, the maximal payoff in the game. ◀

3 A couple more exercises

Another two-by-two game

(i) Find all Nash equilibria of the game shown in the left table below, and verify that in the mixed equilibrium $Eu_1 = 3$ and $Eu_2 = 22/6$.

	L	R
U	$4, 3$	$2, 5$
D	$5, 4$	$1, 3$

(ii) How much better can we do in correlated equilibrium? We wish to maintain the Nash equilibrium payoff ratio $Eu_1/Eu_2 = 9/11$. Let us first see what one can achieve with a simple two-state probability space on the pure Nash equilibria UR, DL with $\pi(UR) = q$, where both players observe the outcome of the device. Verify that expected payoffs are approximately 21% higher than in the mixed equilibrium. (*Hint:* the payoff ratio condition is $2q + 5(1-q) = \frac{9}{11}[5q + 4(1-q)]$)

(iii) Consider a more general distribution, of the form $\pi(UL) = p, \pi(UR) = q, \pi(DL) = r = 1 - p - q$ which avoids the bad outcome DR. Can we do even better with such a distribution, still maintaining the above payoff ratio? The answer turns out to be *no*: we obtain $p = 0$ and we are back to the previous case. *Hints:* the no-deviation conditions imply $p \leq 1/4$; the payoff ratio implies $q = \frac{19}{42} - \frac{1}{21}p$; and then player 1's payoff is decreasing in p.

A four-by-four example

Reconsider the following game:

	W	X	Y	Z
A	$3^*, 2^*$	$0, 0$	$0, 0$	$1, 1$
B	$0, 0$	$2^*, 3^*$	$0, 0$	$1, 1$
C	$0, 0$	$0, 0$	$0^*, 0^*$	$-1, -1$
D	$1, 1$	$1, 1$	$-1, -1$	$0, 0$

We have found that the Nash equilibria are the starred pure equilibria plus a mixed equilibrium where 1 plays A and B with probability $p = 3/5$ on A and 2 mixes between W and X with probability $q = 2/5$ on W.

Determine now ρ such that the distribution $\pi(AW) = \rho, \pi(BX) = 1 - \rho$ is a correlated equilibrium. For which of these value the correlated equilibrium is also Nash?

A three-by-three game

Consider the game depicted in the left table below. The pure Nash equilibria are TC, BC, as you can easily check. The highest average payoff is at TL and BR. We want to find correlated equilibria π where $\pi(TL)$ and $\pi(BR)$ are strictly positive. Note that a π concentrated on those two profiles will not do. Indeed in this case if player 2 is told to play L she knows that 1 is playing T so she would deviate to C. Similarly she would deviate if told to play R. So to satisfy her incentive compatibility constraint, when told to play L she should not be sure that 1 plays T, and similarly if she is told to play R. A reasonable candidate family of distributions is in the right table.

	L	C	R
T	$2,2$	$0,3$	$0,0$
M	$1,1$	$-1,-1$	$1,1$
B	$0,0$	$0,3$	$2,2$

	L	C	R
T	$p/2$	0	0
M	$(1-p)/2$	0	$(1-p)/2$
B	0	0	$p/2$

We must check that $L \succcurlyeq_2 C$ when 2 is told L (same restriction as when she is told R); and that $M \succcurlyeq_1 T, B$ when 1 is told M. The latter is true (with indifference) since L and R have then the same conditional probabilities. The former requires that $2\frac{p}{2}+\frac{1-p}{2} \geq 3\frac{p}{2}-\frac{1-p}{2}$, which gives $0 \leq p \leq 2/3$.

A parametric chicken

Reconsider the parametric family on the left below

	C	F
C	$1,1$	ξ,λ
F	λ,ξ	$0,0$

p	$\frac{1-p}{2}$
$\frac{1-p}{2}$	

where $0 < \xi < 1$ and $\lambda > 1$, and assume that $\lambda + \xi < 2$. The latter says that conflict is detrimental, in the sense that average players' payoff is maximal under CC. Consider the distribution on the right above. The no-deviation property for player 1 in correlated equilibrium requires

$$p \cdot 1 + \frac{1-p}{2}\xi \geq \lambda p$$

that is $p \leq \xi/[\xi + 2(\lambda - 1)]$. The same restriction must hold for player 2, so in correlated equilibrium the expected payoff of each player is

$$\pi_C = p + \frac{1-p}{2}\xi + \frac{1-p}{2}\lambda = p + \frac{1-p}{2}(\lambda+\xi) = \frac{1}{2} + p\left(1 - \frac{\lambda+\xi}{2}\right)$$

which is increasing in p; so substituting the highest p value and observing that $(1-p)/2 =$

$(\lambda - 1)/[\xi + 2(\lambda - 1)]$ we have that the highest correlated equilibrium payoff is

$$\pi_C^* = p + \frac{1-p}{2}(\lambda + \xi) = \frac{\xi}{\xi + 2(\lambda - 1)} + \frac{(\lambda + \xi)(\lambda - 1)}{\xi + 2(\lambda - 1)} = \frac{\lambda(\xi + \lambda - 1)}{\xi + 2(\lambda - 1)}.$$

We can compare this with the payoff in the mixed equilibrium (which is the only symmetric Nash equilibrium of the game). We easily find that the mixed equilibrium is given by $p = \xi/(\xi + \lambda - 1)$, resulting in the expected payoff

$$\pi_M = p^2 + p(1-p)(\lambda + \xi) = \cdots = \frac{\lambda \xi}{\xi + \lambda - 1}.$$

Then

$$\frac{\pi_C^*}{\pi_M} = \cdots = 1 + \frac{(\lambda - 1)^2}{\xi^2 + 2\xi(\lambda - 1)}.$$