Optimal Delegation: the Principal-Agent Model

Luigi Balletta and Salvatore Modica*

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Part I

Adverse Selection

In this case...

1 The basic model

The principal delegates production of quantity q with increasing concave[1] value $v(q)$ to an agent with ability θ. The agent can produce q at constant marginal cost inversely proportional to ability, specifically $c(q,\theta) = \theta^{-1}q$. The agent may be efficient $\theta = \overline{\theta}$ or inefficient $\theta = \underline{\theta}$. To delegate production the principal offers the agent a menu of *contracts* among which the agent can choose. Each contract is a pair $\mathfrak{c} = (q,t)$ whereby the principal proposes to the agent to produce q in exchange for a transfer t. If the agent accepts contract $\mathfrak{c}$ the payoffs are $v(q) - t$ for the principal and $t - c(q,\theta)$ for the agent with ability θ. We specify later what is known about θ at contracting time. If the agent does not accept any contract we assume she gets a "reservation utility" of zero. We assume the agent accepts a contract if indifferent between accepting and refusing. Also, it is common knowledge that the fraction of efficient agents is η. Finally, the principal is assumed to be risk neutral.

Notation for the sequel: we let $\underline{g}(\mathfrak{c}) = t - c(q,\underline{\theta})$ and $\overline{g}(\mathfrak{c}) = t - c(q,\overline{\theta})$ be the net payoffs of the two types of agents under contract $\mathfrak{c}$. A central parameter is $\gamma \equiv \underline{\theta}^{-1} - \overline{\theta}^{-1} > 0$, the marginal cost difference.

1.1 First best

Social welfare is taken to be the sum of the payoffs $v(q) - c(q,\theta)$, so given θ the optimal quantity $q(\theta)$ is the unique solution of the "marginal value equal marginal cost" equation $v'(q) = \theta^{-1}$.[2] Observe that concavity of v implies that $q(\overline{\theta}) > q(\underline{\theta})$ - the efficient agent

*University of Naples and Palermo respectively. This just summarizes a selection of the basic results in Laffont-Martimort *The Theory of Incentives,* Princeton University Press, 2002.

[1] We always mean strictly whenever not otherwise specified; all functions have derivatives of all orders.

[2] We shall assume that social welfare is non-negative at $q(\theta)$.

produces more. The optimal quantity $q(\theta)$ is called the "first best" solution. We assume that optimized social welfare is positive for both θ: $v(q(\theta)) - c(q(\theta), \theta) > 0$.

1.2 Symmetric information

Suppose first that both principal and agent know θ when they meet. Then the principal can propose to agent θ the single contract $(q(\theta), t(\theta))$ where $t(\theta) = c(q(\theta), \theta)$. The agent gets zero payoff so she accepts (she is indifferent) and the first best is produced.

1.3 Asymmetric information before contract

At the other extreme suppose now that when principal and agent meet only the agent knows his "type" θ. This is the strongest possible form of asymmetry. To examine what the principal should do in this situation first observe that under any contract $\mathfrak{c} = (q, t)$ the efficient agent gets a higher payoff than the inefficient one - obviously, since she produces at lower cost. Precisely, you can easily check that $\overline{g}(\mathfrak{c}) = \underline{g}(\mathfrak{c}) + \gamma q$. So from the point of view of the principal the best single contract which is sure to be accepted is the best contract which leaves the inefficient agent with zero payoff; and we know that this is just $(q(\underline{\theta}), t(\underline{\theta}))$, whereby the principal gets $v(q(\underline{\theta})) - c(q(\underline{\theta}), \underline{\theta})$ for sure (the efficient agent is quite happy to accept it). The alternative single contract is one which the inefficient agent rejects, and at that point it is best to leave the efficient agent at zero payoff, that is offer $(q(\overline{\theta}), t(\overline{\theta}))$. This yields the principal expected payoff $\eta \left[v(q(\overline{\theta})) - c(q(\overline{\theta}), \overline{\theta})\right] + (1 - \eta) \cdot 0$.

Either of the two above alternatives may be better than the other, but: can the principal do better than offering a single contract? We shall see that he can do better by offering a *pair* of contracts $\underline{\mathfrak{c}} = (\underline{q}, \underline{t})$ and $\overline{\mathfrak{c}} = (\overline{q}, \overline{t})$, the first designed to be chosen by the agent $\underline{\theta}$ and the second by $\overline{\theta}$. For this plan to go through the following constraints must be satisfied:

$$\overline{g}(\overline{\mathfrak{c}}) \geq \overline{g}(\underline{\mathfrak{c}}), \quad \underline{g}(\underline{\mathfrak{c}}) \geq \underline{g}(\overline{\mathfrak{c}}) \tag{ICC}$$

$$\overline{g}(\overline{\mathfrak{c}}) \geq 0, \quad \underline{g}(\underline{\mathfrak{c}}) \geq 0. \tag{PC}$$

The first two, called *incentive compatibility constraints,* say that each should choose the right contract; the second two, called *participation constraints,* say that each should get non-negative payoff by so doing. The set of constraints make the pair *incentive feasible.* The expected payoff of the principal becomes

$$\begin{aligned} &\eta \left[v(\overline{q}) - \overline{t}\right] + (1 - \eta) \left[v(\underline{q}) - \underline{t}\right] \\ =&\eta \left[v(\overline{q}) - c(\overline{q}, \overline{\theta})\right] + (1 - \eta) \left[v(\underline{q}) - c(\underline{q}, \underline{\theta})\right] - \left[\eta \overline{g}(\overline{\mathfrak{c}}) + (1 - \eta) \underline{g}(\underline{\mathfrak{c}})\right] \end{aligned} \tag{PEP}$$

which is to be maximized under the (ICC) and (PC) constraints. The last expression makes it apparent that the agents' net payoffs are a cost for the principal, and in all cases we have seen the principal was able to keep them down to zero. Can he do it in this case? No: the problem is having the efficient agent pick $\overline{\mathfrak{c}}$, which requires $\overline{g}(\overline{\mathfrak{c}}) \geq \overline{g}(\underline{\mathfrak{c}}) = \underline{g}(\underline{\mathfrak{c}}) + \gamma \underline{q}$. At best the principal can make $\underline{g}(\underline{\mathfrak{c}}) = 0$ and $\overline{g}(\overline{\mathfrak{c}}) = \gamma\underline{q}$. In other words the efficient agent must be granted a payoff of at least $\gamma\underline{q}$ to choose $\overline{\mathfrak{c}}$, otherwise she will pretend she is inefficient and choose $\underline{\mathfrak{c}}$. This is her *informational rent* - which, notice, goes up with the inefficient agent's production.

At $\underline{g}(\underline{\mathfrak{c}}) = 0$ and $\overline{g}(\overline{\mathfrak{c}}) = \gamma\underline{q}$ the constraint $\overline{g}(\overline{\mathfrak{c}}) \geq 0$ is clearly satisfied too. The one we have not considered is $\underline{g}(\underline{\mathfrak{c}}) \geq \underline{g}(\overline{\mathfrak{c}}) = \overline{g}(\overline{\mathfrak{c}}) - \gamma\overline{q}$ which becomes $0 \geq \gamma\underline{q} - \gamma\overline{q}$, satisfied whenever $\overline{q} \geq \underline{q}$. But since the whole point was to get the efficient agent to choose $\overline{\mathfrak{c}}$ and since the cost of this is increasing in $\underline{q}$ we expect $\overline{q} \geq \underline{q}$ to be true at the optimum. By ignoring that constraint, with $\underline{g}(\underline{\mathfrak{c}}) = 0$ and $\overline{g}(\overline{\mathfrak{c}}) = \gamma\underline{q}$ we get the following objective:

$$\begin{aligned}&\eta\left[v(\overline{q}) - c(\overline{q}, \overline{\theta})\right] + (1-\eta)\left[v(\underline{q}) - c(\underline{q}, \underline{\theta})\right] - \eta\gamma\underline{q} \qquad \text{(PEP-REDUCED)}\\ =&\eta\left[v(\overline{q}) - c(\overline{q}, \overline{\theta})\right] + (1-\eta)\left[v(\underline{q}) - \left(\underline{\theta}^{-1} + \frac{\eta}{1-\eta}\gamma\right)\underline{q}\right].\end{aligned}$$

Let us call $\underline{q}^{SB}, \overline{q}^{SB}$ the maximizer of this expression. We will verify that $\underline{q}^{SB} \leq \overline{q}^{SB}$ which says that the neglected constraint is satisfied. In the expression above $\overline{q}$ appears in the term $v(\overline{q}) - c(\overline{q}, \overline{\theta})$ which is maximized by $q(\overline{\theta})$ - so $\overline{q}^{SB} = q(\overline{\theta})$. On the other hand we see that the effective marginal cost of $\underline{q}$ is $\underline{\theta}^{-1} + \frac{\eta}{1-\eta}\gamma$, which is larger than $\underline{\theta}^{-1}$ for it includes the increase in $\overline{g}(\overline{\mathfrak{c}})$ it generates. The value $\underline{q}^{SB}$ is the solution of $v'(\underline{q}) = \underline{\theta}^{-1} + \frac{\eta}{1-\eta}\gamma$ (smaller than $q(\underline{\theta})$ by concavity of v) if the resulting maximum value $v(\underline{q}) - \left(\underline{\theta}^{-1} + \frac{\eta}{1-\eta}\gamma\right)\underline{q}$ is non-negative; otherwise $\underline{q}^{SB} = 0$. Therefore $\underline{q}^{SB} < q(\underline{\theta}) < q(\overline{\theta}) = \overline{q}^{SB}$ so our proviso is true and the original problem is solved. The implied transfers are then $\underline{t} = c(\underline{q}^{SB}, \underline{\theta})$ and $\overline{t} = c(\overline{q}^{SB}, \overline{\theta}) + \gamma\underline{q}^{SB}$.

To sum up: if the agent is efficient she will produce the efficient quantity $q(\overline{\theta})$ but the principal will pay her $\gamma\underline{q}^{SB}$ on top of the production cost $c(q(\overline{\theta}), \overline{\theta})$; and because of this the quantity $\underline{q}^{SB}$ will be less than efficient - so if the agent is inefficient the realized production level is suboptimal. Notice that if positive $\underline{q}^{SB}$ is decreasing in η: a higher probability that the agent is efficient raises the expected informational rent the principal has to pay, and this calls for a reduction in $\underline{q}^{SB}$.

Lastly: is this menu better than a single contract from the point of view of the principal? It is better than offering only $(q(\overline{\theta}), t(\overline{\theta}))$ because the pair $\overline{\mathfrak{c}} = (q(\overline{\theta}), t(\overline{\theta})), \underline{\mathfrak{c}} = (0, 0)$ satisfies (ICC) and (PC) as can be easily checked so it is feasible but not optimal. Offering only $(q(\underline{\theta}), t(\underline{\theta}))$ - that is the pair $\overline{\mathfrak{c}} = (0, 0), \underline{\mathfrak{c}} = (q(\underline{\theta}), t(\underline{\theta}))$ - is not incentive compatible so the incentive compatible solution is better if

$$\eta\left[v(q(\overline{\theta})) - c(q(\overline{\theta}), \overline{\theta}) - \gamma\underline{q}^{SB}\right] + (1-\eta)\left[v(\underline{q}^{SB}) - \underline{\theta}^{-1}\underline{q}^{SB}\right] > v(q(\underline{\theta})) - c(q(\underline{\theta}), \underline{\theta}).$$

For $\eta = 0$ they are equal. Whenever $\underline{q}^{SB} = 0$ the above is positive. If not compute derivative of the left member with respect to η: if positive the above inequality is satisfied. Letting

$q = \underline{q}^{SB}$. The derivative is

$$\begin{aligned}
&\left[v(q(\overline{\theta})) - c(q(\overline{\theta}), \overline{\theta}) - \gamma q\right] - \eta\gamma q' - \left[v(q) - \underline{\theta}^{-1} q\right] + (1-\eta)\left[v'(q) - \underline{\theta}^{-1}\right] q' \\
&= \left[v(q(\overline{\theta})) - c(q(\overline{\theta}), \overline{\theta}) - \gamma q\right] - \left[v(q) - \underline{\theta}^{-1} q\right] \\
&= \left[v(q(\overline{\theta})) - c(q(\overline{\theta}), \overline{\theta})\right] - \left[v(q) - \overline{\theta}^{-1} q\right] > 0
\end{aligned}$$

since $q(\overline{\theta})$ maximizes $v(q) - c(q, \overline{\theta})$. Concisely, we have found the following:

Proposition. *With asymmetric information at contracting time a risk-neutral principal maximizes his expected payoff by offering the incentive feasible two-contract menu* $\underline{\mathfrak{c}} = (\underline{q}^{SB}, \underline{t})$ *and* $\overline{\mathfrak{c}} = (\overline{q}^{SB}, \overline{t})$ *where* $\overline{q}^{SB} = q(\overline{\theta}), \overline{t} = c(q(\overline{\theta}), \overline{\theta}) + \gamma \underline{q}^{SB}$ *and* $0 \leq \underline{q}^{SB} < q(\underline{\theta}), \underline{t} = c(\underline{q}^{SB}, \underline{\theta})$. *In particular* $\underline{g}(\underline{\mathfrak{c}}) = 0$ *and* $\overline{g}(\overline{\mathfrak{c}}) = \gamma \overline{q}^{SB} > 0$.

1.4 Asymmetric information after contract

A weaker form of information asymmetry arises if the parties are both uninformed when they meet, but the agent (and only her) learns θ after an agreement (if any) is reached, before production takes place. We assume the principal offers again a two-contract menu $(\underline{\mathfrak{c}}, \overline{\mathfrak{c}})$. If she accepts then she will know θ when she has to choose between $\underline{\mathfrak{c}}$ and $\overline{\mathfrak{c}}$, so the incentive constraints (ICC) must still hold. However the agent is still uninformed when she has to decide whether to accept the principal's proposal or not, so assuming the pair incentive compatible she will get $\overline{g}(\overline{\mathfrak{c}})$ with probability η and $\underline{g}(\underline{\mathfrak{c}})$ with probability $(1-\eta)$ - a lottery. We assume the agent is an expected utility maximizer with vonNeumann-Morgerstern utility u increasing, weakly concave[3] with $u(0) = 0$. Therefore the proposal will be accepted if it yields non-negative expected utility, that is if

$$\eta u(\overline{g}(\overline{\mathfrak{c}})) + (1-\eta) u(\underline{g}(\underline{\mathfrak{c}})) \geq 0. \tag{PC'}$$

Therefore the principal's objective is to choose $(\underline{\mathfrak{c}}, \overline{\mathfrak{c}})$ to maximize expected payoff (PEP) subject to (ICC) and (PC'). Note that than (PC) is $\underline{g}(\underline{\mathfrak{c}}), \overline{g}(\overline{\mathfrak{c}}) \geq 0$ that is $u(\underline{g}(\underline{\mathfrak{c}})), u(\overline{g}(\overline{\mathfrak{c}})) \geq 0$. So (PC') is weaker than (PC) and the principal is better off in this case - obviously if you think about it, since the informational asymmetry is less pronounced. In particular one may have $u(\underline{g}(\underline{\mathfrak{c}})) < 0$ - but observe that this implies that the inefficient agent is supposed to produce even if it is against her interest, and this can only happen if there is an external court which can enforce the contract.

1.4.1 Risk-neutral agent

In this case the participation constraint (PC') constraint becomes

$$\eta \overline{g}(\overline{\mathfrak{c}}) + (1-\eta) \underline{g}(\underline{\mathfrak{c}}) \geq 0. \tag{PC'-RN}$$

[3] That is strictly concave or linear.

The problem with incentive compatibility is again that it must be $\overline{g}(\overline{\mathfrak{c}}) \geq \overline{g}(\underline{\mathfrak{c}}) = \underline{g}(\underline{\mathfrak{c}}) + \gamma \underline{q}$ but now $\underline{g}(\mathfrak{c}) < 0$ is compatible with the participation constraint. By exploiting this fact the principal can now attain first best payoff $V^* \equiv \eta \left[v(q(\overline{\theta})) - c(q(\overline{\theta}), \overline{\theta})\right] + (1 - \eta)\left[v(q(\underline{\theta})) - c(q(\underline{\theta}), \underline{\theta})\right]$ by having the inefficient agent pay for the information rent of the efficient one, through the following pair:

$$\overline{\mathfrak{c}} = (q(\overline{\theta}), c(q(\overline{\theta}), \overline{\theta}) + (1 - \eta)\gamma q(\underline{\theta})), \quad \underline{\mathfrak{c}} = (q(\underline{\theta}), c(q(\underline{\theta}), \underline{\theta}) - \eta \gamma q(\underline{\theta})). \qquad \text{(RN-FB)}$$

Then $\overline{g}(\overline{\mathfrak{c}}) = (1-\eta)\gamma q(\underline{\theta})$, $\underline{g}(\underline{\mathfrak{c}}) = -\eta\gamma q(\underline{\theta})$ so $\overline{g}(\overline{\mathfrak{c}}) = \underline{g}(\underline{\mathfrak{c}}) + \gamma \underline{q}$ and $\eta \overline{g}(\overline{\mathfrak{c}}) + (1-\eta)\underline{g}(\underline{\mathfrak{c}}) = 0$. And the other incentive constraint $\underline{g}(\underline{\mathfrak{c}}) \geq \underline{g}(\overline{\mathfrak{c}}) = \overline{g}(\overline{\mathfrak{c}}) - \gamma q(\overline{\theta})$ reads $-\eta\gamma q(\underline{\theta}) \geq (1-\eta)\gamma q(\underline{\theta}) - \gamma q(\overline{\theta})$ and is thus satisfied strictly (because $q(\overline{\theta}) > q(\underline{\theta})$).

This solution raises the problem mentioned above: the agent will accept and then refuse to produce if inefficient unless there is a court which forces her to respect the contract.

In this case of risk-neutral agent there is an alternative solution not requiring external intervention, whereby the principal "sells the activity" to the agent at the price $P = V^*$ to be paid in advance. The principal achieves again the first best; the agent accepts because after paying P her best options (as owner) are to produce $q(\overline{\theta})$ if efficient and $q(\underline{\theta})$ if inefficient, so that her ex-ante expected utility is $V^* - P = 0$.

1.4.2 Risk-averse agent

In this case the participation constraint is (PC') in its general form. The outcome we saw in the risk-neutral case cannot be achieved because expected utility - now with concave u- of producing $q(\overline{\theta})$ if efficient and $q(\underline{\theta})$ if inefficient is lower than expected value V^* so that if the principal wants to sell he must do it at a price $P < V^*$. Alternatively he may design an incentive feasible pair $(\underline{\mathfrak{c}}, \overline{\mathfrak{c}})$.

The problem remains in the sense that to meet (PC'), that is to guarantee at lest expected utility $\eta u(\overline{g}(\overline{\mathfrak{c}})) + (1 - \eta)u(\underline{g}(\underline{\mathfrak{c}})) = 0$ the expected payoff $\eta \overline{g}(\overline{\mathfrak{c}}) + (1 - \eta)\underline{g}(\underline{\mathfrak{c}})$ must be *strictly* positive when u is concave - unlike in the case of risk neutrality. So the principal cannot achieve the first best payoff. In fact the principal's payoff will be lower the more the agent is risk averse, and in the limit when risk aversion goes to infinity the (PC') constraint reduces to (PC) (proof of this in the next paragraph). So the case of section 1.3 can be seen as a special case of the present one where the agent is infinitely risk averse.

Details on risk aversion going to infinity (technical). Start with u, assume $u > 0$ and consider the increasing concave transformation $v_\rho = -1/u^\rho$ with $\rho > 0$.[4] Computing derivatives you can check that (with $r(\cdot)$ the Arrow-Pratt index of risk aversion) $r(v_\rho) = r(u) + (1 + \rho)u'/u$ which goes to infinity with ρ. Compute expected utility with v_ρ: letting

[4] If $u < 0$ we would consider $v_\rho = -(-u)^\rho$ with $\rho > 1$. And in the general case we would combine the two. We limit attention to the positive case in the text for simplicity. The whole argument is adapted from note 15 p.484 in MasColell-Whinston-Green *Microeconomic Theory* Oxford University Press.

$x = u(\overline{g}(\bar{\mathfrak{c}}))$ and $y = u(\underline{g}(\underline{\mathfrak{c}}))$ we have

$$Ev_\rho = -\left[\eta x^{-\rho} + (1-\eta) y^{-\rho}\right].$$

The agent represented by Ev_ρ is equally represented by its increasing transformation $-\left[Ev_\rho\right]^{-1/\rho} = \left[\eta x^{-\rho} + (1-\eta)y^{-\rho}\right]^{-1/\rho}$. Assuming $x > y$ we get

$$\lim_{\rho\to\infty} \left[\eta x^{-\rho} + (1-\eta) y^{-\rho}\right]^{-1/\rho} = \lim_{\rho\to-\infty} \left[\eta x^{\rho} + (1-\eta) y^{\rho}\right]^{1/\rho}$$
$$= \lim_{\rho\to-\infty} (1-\eta)^{1/\rho} y \left[\frac{\eta}{1-\eta}\left(\frac{x}{y}\right)^{\rho} + 1\right]^{1/\rho} = 1 \cdot y \cdot [0+1] = y$$

and similarly the limit is x if $x < y$; so $\lim_{\rho\to\infty} \left[\eta x^{-\rho} + (1-\eta)y^{-\rho}\right]^{-1/\rho} = \min\{x, y\}$.

This says that in the limit as $\rho \to \infty$

$$\eta v_\rho(\overline{g}(\bar{\mathfrak{c}})) + (1-\eta) v_\rho(\underline{g}(\underline{\mathfrak{c}}))0 = \min\{\overline{g}(\bar{\mathfrak{c}}), \underline{g}(\underline{\mathfrak{c}})\}$$

which means (PC') becomes (PC). □

2 Applications

2.1 Insurance

In the insurance market there is supply and demand: there are insurance companies who offer insurance and there are consumers who demand insurance - against a possible damage. To simplify things we assume there is only one insurance company (a monopolist) and focus on a fixed damage $d > 0$. On the demand side there are a large number of people, think of there being infinitely many of them. They have differing damage probabilities; and again to keep things simple we assume that a fraction η incurs damage with probability $\overline{\theta}$, the remaining fraction $1-\eta$ with probability $\underline{\theta} < \overline{\theta}$. These probabilities are private information - not observed in particular by the insurance company. If the company provides (possibly partial) insurance to all then *by the Strong Law of Large Numbers* the fraction of customers of type $\overline{\theta}$ will be η with probability 1. Therefore its payoff is that of a risk neutral company facing *a single* customer of type $\overline{\theta}$ (resp. $\underline{\theta}$) with probability η (resp. $1-\eta$). So by the *SLLN* our principal-agent model of section 1.3 applies.

To set the stage we observe preliminary that absent informational asymmetry and assuming consumers are risk averse the company would fully ensure the consumers (of each type) with a sure profit equal to their risk premium (again with probability 1 by the SLLN). Such a transfer of risk from a risk averse to a risk neutral preference we know to be (Pareto) efficient. We shall see that the optimal risk allocation in the presence of information asymmetry does not entail full insurance for all consumers.

The formal model has then the company as risk neutral principal; a contract is now of the form $\mathfrak{c} = (t_a, t_n)$ where t_n is the company's revenue if no accident occurs and t_a is its disbursement in case of accident. The company's aim is to design an optimal incentive feasible menu $(\underline{\mathfrak{c}}, \overline{\mathfrak{c}})$ to maximize its payoff

$$\eta\left[-\overline{\theta}\,\overline{t}_a + (1-\overline{\theta})\overline{t}_n\right] + (1-\eta)\left[-\underline{\theta}\,\underline{t}_a + (1-\underline{\theta})\underline{t}_n\right].$$

The agents are risk averse expected utility maximizers, all with the same vonNeumann u strictly concave with $u(0) = 0$ and the same initial wealth w - they only differ in accident probability which is the one thing the company cannot observe. Under contract $\mathfrak{c} = (t_a, t_n)$ the agent of type θ has expected utility $\theta u(w - d + t_a) + (1 - \theta)u(w - t_n)$. Therefore the type θ incentive compatibility constraint is the following, where primes indicate variables pertaining to $\theta' \neq \theta$:

$$\theta u(w - d + t_a) + (1-\theta)u(w - t_n) \geq \theta u(w - d + t'_a) + (1-\theta)u(w - t'_n).$$

The participation constraints are not exactly as those we have studied above because the agents' reservation utility differ across types: if you think about it, with no insurance the high risk type is worse off than the low risk type, because in that case expected utility is $\omega \equiv \theta u(w-d) + (1-\theta)u(w) = u(w-d) + (1-\theta)\left[u(w) - u(w-d)\right]$ which is clearly decreasing in θ; so $\underline{\omega} > \overline{\omega}$ and type θ participation constraint is

$$\theta u(w - d + t_a) + (1-\theta)u(w - t_n) \geq \omega.$$

To analyze the principal's problem we first change variables: let $h = u^{-1}$ (increasing convex) and let $u_a = u(w - d + t_a),\ u_n = u(w - t_n)$ - the agent's utilities in the two states. Then $t_a = h(u_a) + d - w$ and $t_n = w - h(u_n)$. Substituting these into the objective function and leaving out the terms independent of choice we see that the objective becomes that of *minimizing*

$$\eta\left[\overline{\theta}h(\overline{u}_a) + (1-\overline{\theta})h(\overline{u}_n)\right] + (1-\eta)\left[\underline{\theta}h(\underline{u}_a) + (1-\underline{\theta})h(\underline{u}_n)\right].$$

Next again we guess which constraints are relevant and neglect the others (which we check ex post).[5] Intuition for incentive compatibility emerges fairly easily from the usual question: *who wants to lie?* Clearly here it is the high risk type who would pretend she is low risk. So her incentive compatibility constraint is definitely relevant. Since incentive to lie calls for a positive informational rent we also guess that she is going to have a higher payoff than the low risk agent - whose participation constraint is thus the relevant one. In conclusion the two guessed-relevant constraints are the following, where $\Delta u \equiv \underline{u}_n - \underline{u}_a$:[6]

$$\overline{\theta}\overline{u}_a + (1-\overline{\theta})\overline{u}_n \geq \underline{u}_a + (1-\overline{\theta})\Delta u \qquad \text{(ICC)}$$

$$\underline{u}_a + (1-\underline{\theta})\Delta u \geq \underline{\omega}. \qquad \text{(PC)}$$

[5]Actually in this case we will check that they hold under some additional assumptions.

[6]If something is not apparent check if the identity $ax + (1-a)y = x + (1-a)(y-x)$ has been applied.

To solve the problem observe first that by convexity of h we have

$$\begin{aligned}\overline{\theta}h(\overline{u}_a) + (1-\overline{\theta})h(\overline{u}_n) &\geq h\left(\overline{\theta}\overline{u}_a + (1-\overline{\theta})\overline{u}_n\right)\\ &\geq h\left(\underline{u}_a + (1-\overline{\theta})\Delta u\right)\end{aligned}$$

which says one cannot do better than $\overline{u}_a = \overline{u}_n = \underline{u}_a + (1-\overline{\theta})\Delta u$ (with (ICC) binding). So at the optimum the high risk agent is fully insured. The question remains whether the low risk is fully insured as well - that is whether $\Delta u = 0$ - and the answer is no. To see it substitute the values just found of $\overline{u}_a$ and $\overline{u}_n$ into the objective function; it becomes

$$\eta h\left(\underline{u}_a + (1-\overline{\theta})\Delta u\right) + (1-\eta)\left[\underline{\theta}h(\underline{u}_a) + (1-\underline{\theta})h(\underline{u}_a + \Delta u)\right]$$

to be minimized subject to (PC). For any Δu (hence also at its optimal value) it is clearly optimal to bring $\underline{u}_a$ to its lowest feasible value, whence (PC) is binding. Substituting again into the objective we obtain

$$\eta h\left(\underline{\omega} - (\overline{\theta} - \underline{\theta})\Delta u\right) + (1-\eta)\left[\underline{\theta}h(\underline{\omega} - (1-\underline{\theta})\Delta u) + (1-\underline{\theta})h(\underline{\omega} + \underline{\theta}\Delta u)\right]$$

and equating to zero the derivative of this expression with respect to Δu we obtain

$$\frac{\eta(\overline{\theta} - \underline{\theta})}{(1-\eta)\underline{\theta}(1-\underline{\theta})}h'\left(\underline{\omega} - (\overline{\theta} - \underline{\theta})\Delta u\right) = h'(\underline{\omega} + \underline{\theta}\Delta u) - h'(\underline{\omega} - (1-\underline{\theta})\Delta u)$$

which implicitly defines the optimal Δu. Now the left member is positive so the right one must be positive as well; then convexity of h implies that $\underline{\omega} + \underline{\theta}\Delta u > \underline{\omega} - (1-\underline{\theta})\Delta u$ that is the optimal $\Delta u > 0$. The low risk agent is thus not fully insured, $\overline{u}_a < \overline{u}_n$ strictly. As we will detail below the neglected constraints are satisfied for $\overline{\theta} - \underline{\theta}$ small enough and d large enough.

The impression one gets is that the low risk people are made worse off by the very presence of the high risk types but that is not the case: with monopoly they are brought down to their reservation level anyway. In markets with competitive insurance firms the bargaining power is effectively in the hands of the consumers, and in that case the above intuition is true.[7]

The neglected constraints (technical). blah blah

2.2 The efficiency-equity tradeoff

We consider a population of agents who are to be assigned a production task, each again with linear cost. A fraction η is efficient - marginal cost $\overline{\theta}^{-1}$ - and the rest is inefficient - marginal cost $\underline{\theta}^{-1}$. A contract $\mathfrak{c} = (q, t)$ gives net payoffs $\underline{g}(\mathfrak{c}) = t - \underline{\theta}^{-1}q$ and $\overline{g}(\mathfrak{c}) = t - \overline{\theta}^{-1}q$ to the two types; and again we let $\gamma = \underline{\theta}^{-1} - \overline{\theta}^{-1}$ denote the cost differential. But now

[7] We cover competitive markets in the next part.

the principal is a benevolent planner who cares about the agents' payoffs, according to an increasing concave function $w(g)$; his objective is to maximize expected payoff

$$\eta w(\overline{g}) + (1-\eta) w(\underline{g}). \tag{PEP}$$

Production has increasing concave value $v(q)$ as before so an assignment $(\underline{q}, \overline{q})$ generates total value $\eta v(\overline{q}) + (1-\eta)v(\underline{q})$; this is what the principal can distribute to the agents. So a pair of contracts $\underline{\mathfrak{c}} = (\underline{q}, \underline{t})$, $\overline{\mathfrak{c}} = (\overline{q}, \overline{t})$ must satisfy the *budget constraint*

$$\eta \overline{t} + (1-\eta)\underline{t} \leq \eta v(\overline{q}) + (1-\eta) v(\underline{q}).$$

Let us start with the complete information benchmark: types are known to the principal. Then the problem is to chose the pair $\underline{\mathfrak{c}}, \overline{\mathfrak{c}}$ to maximize (PEP) subject to the budget constraint. There are no other constraints, agents have no choice but to produce the assigned q in exchange for t. In terms of agents' payoffs we can write the constraint as $\eta(\overline{g} + \overline{\theta}^{-1}\overline{q}) + (1-\eta)(\underline{g} + \underline{\theta}^{-1}\underline{q}) \leq \eta v(\overline{q}) + (1-\eta)v(\underline{q})$ or

$$\eta \overline{g} + (1-\eta)\underline{g} \leq \eta\left(v(\overline{q}) - \overline{\theta}^{-1}\overline{q}\right) + (1-\eta)\left(v(\underline{q}) - \underline{\theta}^{-1}\underline{q}\right) \equiv V(\underline{q}, \overline{q}) \tag{BC}$$

where V is aggregate net value. The maximum of V, call it V^* is attained at the first best values $\underline{q} = q(\underline{\theta}), \overline{q} = q(\overline{\theta})$; and by concavity of w we have $\eta w(\overline{g}) + (1-\eta)w(\underline{g}) \leq w(\eta\overline{g} + (1-\eta)\underline{g}) \leq w(V^*)$. Hence the optimum is $\overline{g} = \underline{g} = V^*$ (which satisfies (BC) with equality). Note that the optimum prescribes redistribution from the efficient to the inefficient agents: since $v(q(\underline{\theta})) - \underline{\theta}^{-1}q(\underline{\theta}) < V^* < v(q(\overline{\theta})) - \underline{\theta}^{-1}q(\overline{\theta})$ we have $\overline{t} = V^* + \overline{\theta}^{-1}q(\overline{\theta}) < v(q(\overline{\theta}))$ and $\underline{t} = V^* + \underline{\theta}^{-1}q(\underline{\theta}) > v(q(\underline{\theta}))$. The feasible pair $\overline{t} = v(q(\overline{\theta}))$ and $\underline{t} = v(q(\underline{\theta}))$ - giving each the value she produces - is not optimal, because concavity of w means the principal is concerned about equity. And with complete information he can reach the best he can hope in this dimension - zero inequality - while at the same time having agents produce efficiently - net aggregate production value is at its maximum.

We will now see that if the principal cannot observe the agents' types the two objectives - equity and efficiency - are not compatible: aversion to inequality in agents' well being (concavity of w) will generate inefficiency in production. With asymmetric information the agents can choose between $\underline{\mathfrak{c}}$ and $\overline{\mathfrak{c}}$ since the principal cannot observe directly whether they choose the contract designed for them, hence the pair $(\underline{\mathfrak{c}}, \overline{\mathfrak{c}})$ must also satisfy the incentive compatibility constraints

$$\overline{g}(\overline{\mathfrak{c}}) \geq \overline{g}(\underline{\mathfrak{c}}), \quad \underline{g}(\underline{\mathfrak{c}}) \geq \underline{g}(\overline{\mathfrak{c}}).$$

Observe that equating agents' payoffs does not satisfy the efficient agent constraint: with the complete information contracts $\overline{t} - \overline{\theta}^{-1}q(\overline{\theta}) = V^* = \underline{t} - \underline{\theta}^{-1}q(\underline{\theta}) < \underline{t} - \overline{\theta}^{-1}q(\underline{\theta})$, in other words the efficient agents would pretend she is inefficient. Then as usual we guess that hers is the relevant IC constraint, ignore the other and check that it is satisfied at the optimum.

The problem is then to maximize (PEP) subject to (BC) and $\overline{g}(\overline{\mathfrak{c}}) \geq \overline{g}(\underline{\mathfrak{c}})$; since as before

$\overline{g}(\underline{\mathfrak{c}}) = \underline{g}(\underline{\mathfrak{c}}) + \gamma\underline{q}$ the constraint is

$$\Delta g \equiv \overline{g}(\overline{\mathfrak{c}}) - \underline{g}(\underline{\mathfrak{c}}) \geq \gamma\underline{q}. \tag{ICC}$$

From this we see immediately that eliminating inequality is not feasible. To proceed observe that (BC) must clearly hold with equality; we can re-write it as $\overline{g} = V(\underline{q}, \overline{q}) + (1-\eta)\Delta g$; so (given production decisions) along the constraint $d\overline{g}/d\Delta g = (1-\eta)$. On the other hand the objective is $\eta w(\overline{g}) + (1-\eta)w(\overline{g} - \Delta g)$ so its derivative with respect to Δg is

$$\eta w'(\overline{g})(1-\eta) + (1-\eta)w'(\overline{g} - \Delta g)\left[(1-\eta) - 1\right] = \eta(1-\eta)\left[w'(\overline{g}) - w'(\overline{g} - \Delta g)\right] < 0 \tag{$\star$}$$

by concavity of w. Therefore also the (ICC) constraint is binding: concavity of w makes optimal inequality reduced at the minimum possible level. We have reduced the problem to

$$\max \ \eta w(\overline{g}) + (1-\eta)w(\overline{g} - \gamma\underline{q}) \quad \textit{subject to} \quad \overline{g} = V(\underline{q}, \overline{q}) + (1-\eta)\gamma\underline{q}.$$

Differentiating with respect to $\overline{q}$ (recalling the expression of $V(\underline{q}, \overline{q})$) leads directly to conclude that optimal $\overline{q} = q(\overline{\theta})$. Equating to zero the derivative with respect to $\underline{q}$ is a little longer; after simplification it gives the following expression by which optimal $\underline{q}$ is defined:[8]

$$v'(\underline{q}) = \underline{\theta}^{-1} + \frac{\gamma\eta\left[w'(\underline{g}) - w'(\overline{g})\right]}{\eta w'(\overline{g}) + (1-\eta)w'(\underline{g})} > \underline{\theta}^{-1}$$

which implies $\underline{q} < q(\underline{\theta})$. The conclusion is that with asymmetric information the concern for equity is harmful for efficiency. There is a true (and difficult) tradeoff. Note that if w were *not* concave, say linear as $w(g) = g$, the crucial step where we derived that (ICC) was binding - see ($\star$) - would not go through: the principal's objective $\eta\overline{g} + (1-\eta)\underline{g} = \eta\left(\overline{t} - \overline{\theta}^{-1}\overline{q}\right) + (1-\eta)\left(\underline{t} - \underline{\theta}^{-1}\underline{q}\right)$ would be maximized by efficient production and $\overline{t} = v(q(\overline{\theta}))$, $\underline{t} = v(q(\underline{\theta}))$ (each getting what she produces).[9] The efficient agents' incentive compatibility is satisfied since $v(q(\overline{\theta})) - \overline{\theta}^{-1}q(\overline{\theta}) \geq v(q(\underline{\theta})) - \overline{\theta}^{-1}q(\underline{\theta})$ because $q(\overline{\theta})$ maximizes $v(q) - \overline{\theta}^{-1}q$. Quite possibly $\Delta g > \gamma q(\underline{\theta})$ strictly, but this is no concern in this case.

[8] Recalling that $V(\underline{q}, \overline{q}) = \eta\left(v(\overline{q}) - \overline{\theta}^{-1}\overline{q}\right) + (1-\eta)\left(v(\underline{q}) - \underline{\theta}^{-1}\underline{q}\right)$ and that form the constraint $\partial\overline{g}/\partial\underline{q} = (1-\eta)\left(v'(\underline{q}) - \underline{\theta}^{-1} + \gamma\right)$ the derivative of the objective function with respect to $\underline{q}$ reads

$$\begin{aligned}
&\eta w'(\overline{g})\frac{\partial\overline{g}}{\partial\underline{q}} + (1-\eta)w'(\overline{g} - \gamma\underline{q})\left(\frac{\partial\overline{g}}{\partial\underline{q}} - \gamma\right) \\
=&\eta(1-\eta)w'(\overline{g})\left(v'(\underline{q}) - \underline{\theta}^{-1} + \gamma\right) + (1-\eta)w'(\underline{g})\left((1-\eta)\left(v'(\underline{q}) - \underline{\theta}^{-1} + \gamma\right) - \gamma\right) \\
=&(1-\eta)\left(v'(\underline{q}) - \underline{\theta}^{-1}\right)\left[\eta w'(\overline{g}) + (1-\eta)w'(\underline{g})\right] - \gamma\eta(1-\eta)\left(w'(\underline{g}) - w'(\overline{g})\right)
\end{aligned}$$

where we use $\overline{g} - \gamma\underline{q} = \underline{g}$. Equating this expression to zero gives the equation in the text. You can believe that at the optimum the neglected constraint holds.

[9] The argument is true a fortiori for convex w since then $w(\eta\overline{g} + (1-\eta)\underline{g}) < \eta w(\overline{g}) + (1-\eta)w(\underline{g})$.

Part II

Moral Hazard

blah blah

1 The basic model

We study a simple 2×2 model where production $q \in \{\underline{q}, \overline{q}\}$ - with $\underline{q} < \overline{q}$ - depends stochastically on the level of effort $e \in \{0, 1\}$ exerted by the agent, in the sense that if $e = 0$ the probability of $\overline{q}$ is π_0 while if $e = 1$ it is $\pi_1 > \pi_0$. The informational asymmetry here is that the principal can observe the result q but not the effort level e, so if he wants to delegate production to the agent his payment t can only depend on q - in this model it has to be a transfer scheme $(t(\underline{q}), t(\overline{q})) \equiv (\underline{t}, \overline{t})$. Value of production $v(q)$ can be $\underline{v} = v(\underline{q})$ or $\overline{v} = v(\overline{q})$ so the principal's expected payoff if $e = i \in \{0, 1\}$ is

$$V_i \equiv \pi_i \left[\overline{v} - \overline{t}\right] + (1 - \pi_i) \left[\underline{v} - \underline{t}\right]$$

Let B_i denote the expected value from effort i: $B_i = \pi_i \overline{v} + (1 - \pi_i)\underline{v} = \underline{v} + \pi_i \Delta v$, where $\Delta v = \overline{v} - \underline{v}$; and let also $\Delta\pi = \pi_1 - \pi_0$; then the increase in expected value that effort generates $\Delta B = B_1 - B_0$ is given by

$$\Delta B = \Delta\pi \Delta v.$$

As we can guess inducing effort will also have a cost, so the principal's decision will be based on comparison of the above marginal benefit with the marginal cost of inducing effort.

The agent has vonNeumann utility $u(t) - \psi(e)$ where $\psi(0) = 0 < \psi(1) = \psi$ and u is increasing concave with $u(0) = 0$. Under contract $(\underline{t}, \bar{t})$ if $e = i$ the probability of $\bar{t}$ is π_i so letting $\overline{u} = u(\bar{t})$ and $\underline{u} = u(\underline{t})$ she will exert (positive) effort if

$$\pi_1 \overline{u} + (1 - \pi_1)\underline{u} - \psi \geq \pi_0 \overline{u} + (1 - \pi_0)\underline{u}$$

so if the principal wishes to induce her to do so he must choose a pair $(\underline{t}, \bar{t})$ which satisfies the above incentive compatibility constraint, and also the participation constraint

$$\pi_1 \overline{u} + (1 - \pi_1)\underline{u} - \psi \geq 0. \tag{PC}$$

A contract satisfying the two constraints is called (as in Part I) *incentive feasible.* If such a contract is signed the agent will exert effort. The principal may on the other hand prefer to offer a different contract, whereby production will be obtained with $e = 0$. We shall see on what the choice depend.

The above incentive compatibility constraint just says that the increase in expected utility that effort generates must be higher than marginal effort cost ψ. This is apparent we re-write it by letting $\Delta u = \overline{u} - \underline{u}$; it then reads

$$\Delta\pi\Delta u \geq \psi. \tag{ICC}$$

Observe that since $V_i = B_i - \left[\pi_i \overline{t} + (1-\pi_i)\underline{t}\right]$ and B_i is independent of the contract, maximization of V_i amounts to minimization of the cost $\pi_i \overline{t} + (1-\pi_i)\underline{t}$ subject to (ICC) and (PC). we shall use this fact repeatedly in the sequel.

1.1 The complete information benchmark

Suppose that effort is observable and that there is a court that can enforce contracts. Then if the principal wishes the agent to exert effort he has to choose $(\underline{t}, \overline{t})$ to maximize V_1 subject to (PC) *only* - since once the agent accepts she must exert effort. As we observed above the problem is to minimize the cost of inducing effort $\pi_1 \overline{t} + (1-\pi_1)\underline{t}$. Letting $h = u^{-1}$ we have to minimize $\pi_1 \overline{t} + (1-\pi_1)\underline{t} = \pi_1 h(\overline{u}) + (1-\pi_1)h(\underline{u})$ subject to $\pi_1 \overline{u} + (1-\pi_1)\underline{u} = \psi$ in the variables $(\underline{u}, \overline{u})$. Since u is concave h is convex so $\pi_1 h(\overline{u}) + (1-\pi_1)h(\underline{u}) \geq h(\pi_1 \overline{u} + (1-\pi_1)\underline{u}) = h(\psi)$; therefore the minimum is attained at $\overline{u} = \underline{u} = \psi$. In terms of transfers we have found that the optimal contract is $\underline{t} = \overline{t} = h(\psi)$. The transfer is thus independent of output: the risk neutral principal bears all the risk, in other words *the agent is fully insured,* which is efficient since the agent is risk averse. At the optimum the cost of inducing effort $C^{FB} = h(\psi)$, and

$$V_1^{FB} = \underline{v} + \pi_1 \Delta v - h(\psi).$$

If the principal wishes to have the agent exert zero effort he can just set $\underline{t} = \overline{t} = 0$ since then the (PC) constraint $\pi_0 \overline{u} + (1-\pi_0)\underline{u} \geq 0$ holds with equality. Therefore

$$V_0^{FB} = \underline{v} + \pi_0 \Delta v.$$

We then have $V_1^{FB} \geq V_0^{FB}$ if and only if $\Delta\pi\Delta v = \Delta B \geq h(\psi)$. Since here $h(\psi)$ is the marginal cost of inducing effort this just says that the principal will induce the agent to exert effort if the marginal benefit of doing so is larger than its marginal cost. *We shall assume in the sequel that* $\Delta B \geq h(\psi)$ so that in the first best the principal induces positive effort.

1.2 Risk neutral agent

Again we start with the case of risk neutral agent. Then (up to a linear transformation) $u(t) = t$ so that $\Delta u = \Delta t$. To induce effort the principal has to choose a contract to maximize V_1 subject to (ICC) and (PC). Under current hypothesis this amounts to minimize $\pi_1 \overline{t} + (1-\pi_1)\underline{t} = \underline{t} + \pi_1 \Delta t$ subject to (ICC) $\Delta\pi\Delta t \geq \psi$ that is $\Delta t \geq \psi/\Delta\pi$ and (PC) $\underline{t} + \pi_1 \Delta t \geq \psi$. Therefore clearly the minimum is attained by having both constraints hold with equality. The principal's payoff is $B_1 - \psi$, and the implied transfers are

$$\underline{t} = -\pi_0 \frac{\psi}{\Delta\pi}, \quad \bar{t} = (1-\pi_0)\frac{\psi}{\Delta\pi}.$$

As in section 1.4.1 we have again the liability problem $\underline{t} < 0$; also since $\pi_1 \bar{t} + (1-\pi_1)\underline{t} = \psi$ says that the principal pays exactly the agent's marginal cost of effort we see that effort is induced whenever $\Delta B > \psi = h(\psi)$ that is whenever it is induced in the complete information case, in other words again the first best is attained under incomplete information if the agent is risk neutral.

Finally, again as in 1.4.1 the first best can be implemented by selling the activity to the agent at price $P = B_1 - \psi$. The principal gets the same payoff as before; and whenever $\Delta B > \psi$ the agent as owner will find it optimal to exert effort, and her expected payoff is then $B_1 - \psi = P$ so she will accept to buy. But note that under this arrangement she will bear the risk that output is low in which case her realized payoff is $\underline{v} - B_1 < 0$. This is acceptable as long as she is risk neutral, but it will not be optimal if risk averse. We shall return to this next.

1.3 Risk averse agent

To induce effort the problem now is to minimize $\pi_1 \bar{t} + (1-\pi_1)\underline{t} = \pi_1 h(\bar{u}) + (1-\pi_1)h(\underline{u})$ subject to (PC) $\underline{u} + \pi_1 \Delta u \geq \psi$ *and* (ICC) $\Delta u \geq \psi/\Delta\pi$ (recall that in the complete information case of section 1.1 (ICC) was not an issue). Since $\bar{u} = \underline{u} + \Delta u$, for given Δu clearly we want (PC) binding. The only issue is whether it is worth to raise Δu above $\psi/\Delta\pi$. With (PC) binding the payoff becomes

$$\pi_1 h(\psi + (1-\pi_1)\Delta u) + (1-\pi_1)h(\psi - \pi_1 \Delta u)$$

whose derivative with respect to Δu is

$$\begin{aligned} &\pi_1(1-\pi_1)h'(\psi + (1-\pi_1)\Delta u) - \pi_1(1-\pi_1)h'(\psi - \pi_1\Delta u) \\ =&\pi_1(1-\pi_1)\left[h'(\psi + (1-\pi_1)\Delta u) - h'(\psi - \pi_1 \Delta u)\right] > 0 \end{aligned}$$

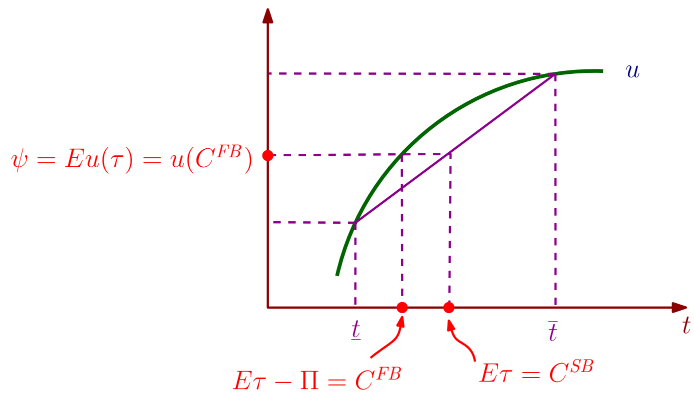
by convexity of h. So raising Δu makes the cost increase, whence $\Delta u = \psi/\Delta\pi$ at the optimum. Upon substitution we get the solution - use SB to identify it -

$$\underline{t}^{SB} = h\left(-\frac{\pi_0}{\Delta\pi}\psi\right), \quad \bar{t}^{SB} = h\left(\frac{1-\pi_0}{\Delta\pi}\psi\right).$$

An important point to note is that $\underline{t}^{SB} < \bar{t}^{SB}$, that is the agent is not fully insured. At the optimum the cost of inducing effort is given by (inequality from convexity of h)

$$\begin{aligned} C^{SB} &= \pi_1 \bar{t}^{SB} + (1-\pi_1)\underline{t}^{SB} = \pi_1 h\left(\frac{1-\pi_0}{\Delta\pi}\psi\right) + (1-\pi_1) h\left(-\frac{\pi_0}{\Delta\pi}\psi\right) \\ &> h\left(\pi_1 \frac{1-\pi_0}{\Delta\pi}\psi - (1-\pi_1)\frac{\pi_0}{\Delta\pi}\psi\right) = h(\psi) = C^{FB}. \end{aligned}$$

The difference $C^{SB} - C^{FB}$ is exactly the agent's risk premium for the lottery $\tau \equiv (\underline{t}^{SB}, 1 - \pi_1; \bar{t}^{SB}, \pi_1)$. Indeed $C^{SB} = E\tau$; (PC) binding means $Eu(\tau) = \underline{u} + \pi_1 \Delta u = \psi$; and the risk premium Π of τ is defined by $u(E\tau - \Pi) = Eu(\tau)$. So $u(C^{SB} - \Pi) = \psi$; but $h(\psi) = C^{FB}$ means $u(C^{FB}) = \psi$, whence $C^{FB} = C^{SB} - \Pi$ or $\Pi = C^{SB} - C^{FB}$. Illustration below.



The cost of inducing zero effort is still zero and the benefit still B_0. So effort will be induced if $\Delta B \geq C^{SB}$: with risk averse agent the condition on ΔB is more stringent.

1.4 Summing up

We record the main finding of this section in the following

Proposition. *In any case the agent ends up with zero expected payoff. With complete information the principal has cost $C^{FB} = u^{-1}(\psi)$ of inducing effort, and he does so if $\Delta B \geq C^{FB}$. If this is the case the agent is fully insured, gets paid C^{FB} independently of outcome. With incomplete information, if the agent is risk neutral effort can be induced at the same cost C^{FB} so effort is implemented under the same condition as in the complete information case. If on the other hand the agent is risk averse the cost of inducing effort ls larger, $C^{SB} > C^{FB}$ so that the implementation condition $\Delta B \geq C^{SB}$ is more stringent. If satisfied the agent bears some risk, $\underline{t}^{SB} < \underline{t}^{SB}$, and the difference $C^{SB} - C^{FB} = \Pi$ where Π is her risk premium for the lottery $(\underline{t}^{SB}, 1 - \pi_1; \bar{t}^{SB}, \pi_1)$.*

2 Applications

2.1 Sharecropping

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2.2 Financial contracts

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2.3 Insurance

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