Cooperating Through Leaders*

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Abstract

We model conflict as a prisoner's dilemma between two groups represented by leaders: two group leaders who share group preferences and a common leader concerned with overall welfare. Leaders make recommendations and promises, and face penalties for unfulfilled commitments. A common leader is accountable when the penalty is large enough, in which case she induces cooperation if she moves last. When she does not, cooperation can still emerge with high probability provided the group leaders face large penalties for broken promises. The model highlights accountability and timing as key mechanisms sustaining cooperation between groups.

Keywords: Social Conflict, Polarization, Accountability, Political Equilibria.

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1. Introduction

In 1905 the Russo-Japanese War ended with the Treaty of Portsmouth thanks to the intervention of the U.S. President Theodore Roosevelt, whose efforts were rewarded with the Nobel Peace Prize the following year. Yet the intervention of a third party does not always succeed. Doyle and Sambanis (2006) examine 121 civil wars between 1945 and 1999: 99 concluded with a military victory or a truce - that is, without a successful external intervention - while only 14 ended with a settlement, achieved with the help of the UN. Under what conditions can external agents foster cooperation between two conflicting groups? This is the central question of the paper.

We address this by modeling the intervention of a third party concerned with the well-being of both sides, whom we call the "common leader", in a conflict game between two groups. Alongside the natural leaders of the two groups, this figure acts as a third player. Since in practice leaders conduct negotiations and make strategic choices, we assume that group members evaluate leaders' proposals and act accordingly. In effect, the groups play the game through their leaders, and the strategic interaction we analyze is the game among leaders.

The starting point is a prisoners' dilemma between two groups, with choices of cooperation (C) or fighting (F). The cooperative outcome CC yields 1 to both groups, but left unaided the two groups will fight - that is play FF - and get zero. We enrich this framework by introducing leaders, who recommend courses of action to the groups. Leaders act to influence the outcome of the game because their own utility depends on that outcome. Groups act as followers: they adopt the most promising proposal, but punish leaders who fail to deliver on their promises.

As indicated above we distinguish two types of leaders. *Group* leaders share the payoffs of their own groups, while the *common* leader values the average welfare of both groups. Thus the leaders' game has three players: two partisan leaders and one common leader. The leaders propose action profiles in the underlying game, which may be interpreted "recommendations and promises". Group leaders address only their own groups, while the common leader can address both. To illustrate, suppose the common leader proposes CC - which is interpreted as "cooperate; the other group will cooperate too". If group leader 2 proposes FF, group 2 will follow the common leader's offer (promising 1 versus zero) and play C. If group leader 1 is proposing FC - "fight; the other side will yield", which gives group 1 the highest payoff - group 1 will choose this over CC and will play F. The outcome will then be FC. Both groups and their leaders obtain the FC payoff, and the common leader is punished by group 2 for failing to deliver the promised utility of 1 (at FC they get even less than zero).

Note that in our setup leaders do not act as substitutes for their citizens in the choice of the action. Rather, their role is to shape expectations and thereby coordinate the actions of individuals, who ultimately retain the power of choice. Leaders exercise this role by presenting a plan, that is a proposed course of action for the entire population. For such a plan to be effective it must appeal to the group members, so that they are willing to adopt it, and at the same time it has to satisfy the feasibility constraints implicit in the game, ensuring that the promised outcomes can indeed arise from the action profile suggested by the leader.

We now describe the results of the analysis. Firstly, in the absence of a common leader, in the resulting game between the two group leaders the only equilibrium outcome is fighting, FF. So only if a common leader is present can conflict be avoided.

Assume a common leader is there. Then there are two necessary and sufficient conditions for cooperation to emerge with certainty in equilibrium: the common leader must move last and must suffer adequate punishment if she fails to deliver. Moving last puts the common leader in the best position to induce her most preferred outcome, which is cooperation; but in the absence of adequate punishment she will give in to an aggressive group leader and induce a winner/loser outcome, unbothered by the loser group's reaction. That an accountable common leader should have the last word might be regarded as the main message of the paper.

If either of the above conditions fail cooperation for sure cannot be an equilibrium. A general condition favoring cooperation is then that the groups can inflict large punishments to their group leaders. Specifically, when the common leader does not move last, as the level of the group leaders' punishment goes to infinity there is an equilibrium where the probability of cooperation goes to 1. The intuition here is that the threat of punishment tends to deter group leaders from making excessively optimistic promises - think of the first group leader proposing FC - and this leaves room for a successful CC proposal by the common leader.

Related Literature

The closest antecedent to the present paper is Baliga et al. (2011), who also study 2×2 games with punishable leaders. In their setting, leaders choose actions directly (such as F or C), groups are heterogeneous, and punishments arise when leaders make choices that are not best responses from members' perspectives. Their focus is on political survival. Despite superficial similarities, this framework is substantially different from ours - we emphasize promises and competition among leaders in a model where decisions are ultimately taken by citizens. A related analysis can be found in Block et al. (2025); but in their model leaders choose incentive compatible social norms; they study how the size and heterogeneity of groups impacts on segregation and the intensity of the conflict.

Our specification of leaders' proposals as expectation-shaping builds on Hermalin (1998), where a single leader of one group observes a payoff-relevant signal and, by acting on it, shapes followers' expectations so that imitation is an equilibrium.

We are not the first to highlight potential benefits of third-party intervention. Meirowitz et al. (2019) study a neutral broker with no stake in outcomes who facilitates cooperation by eliciting private information. Our common leader differs because she is directly involved, with utility tied to group welfare.

In political economy it is common to model competing leaders (e.g., Dewan and Squintani (2018)). However the central issue there is how groups with different preferences choose a single leader who decides for all, as in electoral competition. In our setting there is no "winning leader", and different groups may follow different leaders, which is a more appropriate assumption in conflict situations.

Other related work emphasizes delegation and leadership. In Eliaz and Spiegler (2020), a representative agent selects among competing narratives, akin to our groups choosing among proposals. Dutta et al. (2018) study punishment of leaders, but punishment is based on ex ante considerations and there is no common leader. Prat and Rustichini (2003) analyze games among principals mediated by agents who receive transfers to induce specific actions, echoing the role of proposers.

More loosely related is the literature on polarization and conflict. Esteban and Ray (1994), Esteban et al. (2012), and Duclos et al. (2004) develop measures of polarization; Esteban and Ray (2008) analyze the salience of ethnic conflict. These frameworks have been widely tested empirically (e.g., Esteban et al. (2012)).

Finally, our leaders' game shares some features with correlated equilibria in the underlying game: in both, external intervention can improve outcomes over Nash equilibria, and leaders or mediators suggest action profiles. Yet the differences are deeper. In correlated equilibria, the mediator is impartial and not punishable; in our setting, leaders are strategic players and followers can punish them. We provide a formal comparison in Appendix D.

Outline of the Paper

Section 2 sets up the model. Section 3 considers the case where only group leaders are present. Section 4 contains the main result, intuition, and discussion. Section 5 develops the detailed analysis of the models studied. Section 6 concludes. Appendices A to C contain proofs of assertions made in the text; Appendix D compares the leaders equilibria with the correlated equilibria of the game.

2. The Model

2.1. The Underlying Game

The are two homogeneous groups denoted by $k \in \{1, 2\}$, and each group has a representative follower. Follower k chooses action $a_k \in \{C, F\} \equiv A_k$, where C means cooperation and F fight. Action profiles (a_1, a_2) are denoted by $a \in A$, and at a all members of group k receive utility $u_k(a)$. These utility functions give rise to the *underlying game*.

The underlying game on which the paper is focused is the prisoners dilemma. If both followers play C they get a higher utility than if they both play F, and we set $u_k(CC) = 1$ and $u_k(FF) = 0$ for both k. Also, $u_1(FC) = u_2(CF) = \lambda > 1$ and $u_1(CF) = u_2(FC) = \xi < 0$.[4] The game matrix is thus

	C	F
C	$1, 1$	ξ, λ
F	λ, ξ	$0, 0$

We are only interested in games in which conflict is detrimental, so we assume that the average group payoff is maximum at CC:

$$(\lambda + \xi)/2 < 1. \tag{1}$$

[4]Action profiles are always written so that 1 proceeds 2, and we are omitting commas when possible.

2.2. The Leaders' Games

We now describe the games played by the leaders, which are the object of the analysis. There are three leaders $\ell \in \{0, 1, 2\}$: two *group leaders* $\ell = 1, 2$ who have the same interest as group $k = \ell$, and a *common leader* $\ell = 0$ who cares about both groups. The payoff of the leaders is the sum of a direct component and a possible punishment imposed by the followers.

The direct utility depends on the action profile $a \in A$ played by the followers in the underlying game. Denoting by $U^\ell(a)$ the utility leader ℓ obtains from profile a, we take $U^\ell(a) = u_\ell(a)$ for $\ell = 1, 2$; and we assume that the common leader's preferences coincide with utilitarian welfare: $U^0(a) = (u_1(a) + u_2(a))/2$.[5] We now describe how the profile a played by the followers and the possible punishments to the leaders are determined.

The three leaders make proposals to their potential followers, which may interpreted as *recommendations and promises*. Specifically, each leader makes a proposal $s^\ell \in A$, that is, an action profile in the underlying game. Proposing s^ℓ to group k means recommending the group to play s^ℓ_k and suggesting that the other group plays s^ℓ_{-k}, thus promising utility $u_k(s^\ell)$; in words the leader's proposal is "Follow me: play s^ℓ_k, and you will get utility $u_k(s^\ell)$".

We will consider three different extensive form games, differing in the order of the leaders' moves. We present the games going from the case in which the common leader moves last, to the one in which she moves first. In the first one the two group leaders move first, simultaneously choosing their proposals; their choices are communicated to the common leader, who then chooses her proposal; the triple of the resulting proposals made by the leaders is finally communicated to the followers, who choose their actions in the underlying game - a_1 and a_2 respectively - and thus determine their own payoff and the leaders' direct utility. In the second version the three leaders move simultaneously, and then the game proceeds as in the previous case - the leaders' choices are communicated to the followers who then choose actions C or F in the underlying game. In the third version the common leader moves first, choosing a proposal in A; her choice is communicated to the group leaders, who then simultaneously choose their proposals.

We still have to specify how the followers choose actions given a profile of proposals by the three leaders, and how the payoffs at final nodes are determined; to this we turn. Observe that in all the extensive forms games just introduced the strategies of the leaders result in a triple of proposals communicated to the followers, which we denote by $s \equiv (s^0, s^1, s^2) \in A \times A \times A$. We assume that the follower of group k considers the proposal of the corresponding group leader and the one by the common leader; in other words follower k considers s^k and s^0.[6] Among the proposals they consider, the followers choose the one promising them the highest utility; and if follower k chooses $s^\ell = a$ then group k plays a_k, expecting $u_k(a)$. More precisely, given a triple s follower k chooses the proposal that maximizes $u_k(s^\ell)$ over the proposals s^0 and s^k she considers. Denote the chosen

[5] We use superscripts for leaders and subscripts for followers.

[6] As a benchmark we will analyze in Section 3 the case in which followers ignore the proposal of the common leader; in this case each group just follows their own group leader.

proposal by $g^k(s) \in A$.[7] Having chosen $g^k(s)$ group k then play their part $g^k(s)_k$, expecting to get $u_k(g^k(s))$. Therefore, given a triple (s^0, s^1, s^2) the *implemented action profile* in the underlying game will be $g(s) \equiv \left(g^k(s)_k\right)_{k=1,2} \in A$. This determines the utility of the groups, $u_k(g(s))$, and the direct utility of the leaders $U^\ell(g(s))$. If for example $s^0 = FC, s^1 = FC, s^2 = CF$ then $g^1(s) = FC$ and $g^2(s) = CF$ so both groups will play F, and $g(s) = FF$. Note that follower 1 is complying with the recommendations of both $\ell = 0$ and $\ell = 1$. Of course in this case no leader fulfills her promise (because all have promised $\lambda > 1$ but the realized utility is 0).

As to punishments, if a leader's promise is not fulfilled she will be punished by the groups who have complied with their proposals. Precisely, each group has the ability to impose a utility penalty $P > 0$ on their group leader, and $Q/2 > 0$ on the common leader (who then loses Q if punished by both groups). And if $u_k(g^k(s)) < u_k(g(s))$ then group k punishes any leader $\ell \in \{0, k\}$ such that $s^\ell = g^k(s)$, where the punishment is P if $\ell = k$ and $Q/2$ if $\ell = 0$. In the example above group 1 punishes leaders $\ell = 0$ and $\ell = 1$, and group 2 punishes $\ell = 2$.

Finally we define the payoffs. In all the three extensive forms, given the leaders' strategies, their payoffs - direct utility and punishments - depend only on the triple $s = (s^0, s^1, s^2)$ of proposals that are communicated to the followers. Denoting by $V^\ell(s)$ the payoff of leader ℓ, and letting $\mathbf{1}\{\mathfrak{c}\} = 1$ if condition $\mathfrak{c}$ is true and zero otherwise, the payoff of a group leader $\ell = 1, 2$ is

$$V^\ell(s) = U^\ell(g(s)) - P \cdot \mathbf{1}\{\ell = k \ \&\ g^k(s) = s^\ell \ \&\ u_k(s^\ell) < u_k(g(s))\} \tag{2}$$

and the payoff of the common leader is

$$V^0(s) = U^0(g(s)) - (Q/2) \cdot \sum\nolimits_{k=1,2} \mathbf{1}\{g^k(s) = s^0 \ \&\ u_k(s^0) < u_k(g(s))\}. \tag{3}$$

The games played by the three leaders which we have defined will be referred to as *leaders games*. The solution concept we adopt is a strengthening of the subgame perfect equilibrium: we require that in each subgame the leaders play a Nash equilibrium in weakly undominated strategies. The games are finite, so subgame perfect equilibria in mixed strategies exist. We call these *leaders equilibria*. We are interested in the conditions under which the implemented profile in the equilibria of the leaders game is the cooperative outcome, at least with positive probability.

Comment on the role of groups. Regarding the role of group members as passive followers, the point is that expectations can be *shaped* by leaders even under full information. Groups may believe that leaders are better at predicting behavior and thus follow their advice. This captures both the influence of charismatic leaders and the reality that in large groups individuals may find it costly to acquire full information, preferring to delegate strategic assessments while retaining the option to punish non-delivering leaders ex post.

[7] The maximizer $g^k(s)$ for group k is unique because $a \neq a'$ implies $u_k(a) \neq u_k(a')$ for both k, though it may be proposed by more than one leader.

3. Nothing Is Gained With Only Group Leaders

We first establish that if each group only considers proposals from their own group leader the outcomes of the leaders game are the same as in the underlying game. This is in fact true quite generally, that is for any game with any number of groups:

Proposition 1. *For any leaders game, if each group only considers the proposal of their own group leader then at the Nash equilibria of the leaders game the distributions of action profiles chosen by the groups are the same as those induced by the Nash equilibria of the corresponding underlying game.*

The proof is in Appendix A.

4. The General Picture with a Common Leader

So without a common leader conflict in a prisoners dilemma is unavoidable, and the paper is focused on the possibility of reaching a cooperative outcome with the intervention of a common leader. The main points of the paper are contained in the following:

Theorem (Main Result)**.** *Cooperation for sure obtains in equilibrium if and only if* $Q > \lambda + \xi$ *and the common leader moves last. If either condition fails then, at best, cooperation is achieved with high probability when* P *is large.*

The assertions follow from the analysis carried out in Section 5, which begins with a more detailed summary of the results.

4.1. Some intuition, and the main arguments behind the proofs

The inequality $Q > \lambda + \xi$ is an accountability condition on the common leader. It may seem counter-intuitive that it matters so much: since the common leader wants to maximize joint payoff, then why is it necessary that the punishment she faces is sufficiently high? The answer is that although the common leader prefers cooperation to a winner/loser outcome, when a group leader plays aggressively cooperation is out of the question (for example if leader 1 plays FC group 1 will surely play F); the only two outcomes the common leader can induce are an asymmetric outcome (FC, CF) or FF, and if $Q < \lambda + \xi$ she favors the former, hence will not block the aggressive proposal.

Let us see the argument in the case where the common leader moves last. If $Q < \lambda + \xi$ the common leader is better off at FC and CF, even if punished by the losing group, than at FF; indeed at FF she gets at most zero, while in the former case she gets $(\lambda + \xi - Q)/2 > 0$. So if she can only induce FC or FF she will go for FC. Now suppose leader 1 plays FC (a case in point). Then the common leader can - and will - induce FC (possibly at the cost of being punished by group 2) unless leader 2 plays CF, in which case she cannot avoid the FF outcome. The argument leads to the conclusion that if $Q < \lambda + \xi$ the equilibrium outcomes can only be FC, CF or FF.

Suppose on the other hand that $\lambda + \xi < Q$ - in which case the common leader would rather induce FF than FC plus punishment.[8] In this case FC cannot be part of an equilibrium; for suppose 1 plays FC. Then (since group 1 is playing F) group 2 can get at most zero; and it achieves that bound by playing FF, since if she does so the common leader will induce FF by playing FF. Thus leader 1 ends up being punished if she plays aggressively, and that is suboptimal since FF would guarantee her zero.[9] Excluding aggressive play the group leaders are promising at most 1 to their respective groups, and the common leader will then win both groups and induce cooperation by playing CC. The result is cooperation.

Note that the common leader moving last is a double-edged sword: cooperation for sure if she can be effectively punished, no cooperation for sure if not. These are best and worst outcomes, since when the common leader does not move last, as we hint below cooperation with high probability may occur in equilibrium if P is large enough.

The order of moves matter because when the common leader dos not move last the equilibria are typically mixed, so cooperation with certainty cannot be achieved. Still, equilibria with high probability of cooperation may emerge. To see how consider the simultaneous move game. It turns out that in that game the common leader will only play CC or FF, and the group leaders will only play aggressively (FC and CF respectively) or FF. First observe how pure equilibria may fail to exist. Indeed, take to fix ideas large P and Q and suppose that the common leader plays CC and that leader 1 plays FC; then 2's best reply is FF (with CF she is punished and P is high); but if the group leaders play FC, FF then the common leader's best reply is FF (with CC she is punished by group 2); this is the kind of circularity that leads to mixing. Now among the mixed equilibria there is one where the common leader plays CC for sure and the group leaders mix between aggressive play and FF. If the common leader plays CC, if leader 1 plays FC she gets punished in the event 2 plays CF; and the higher the level of P the higher the expected cost of doing it. Thus for high P both group leaders tend to refrain from playing aggressively (they play FF instead), and cooperation obtains with high probability.

4.2. Discussion

It is not easy to single out a specific game timing as being more realistic than the others. Of course real conflicts and negotiations are far more complex than the simple model we have studied, and hardly ever do some parts have a "last word". Most often, especially in international affairs, one observes many rounds of negotiations, with no player really moving last. In these sometimes rather dramatic situations we tend to view the simultaneous moves game as the most appropriate. On the other hand it is also possible that an influential third party steps in later in the conflict at a

[8] The following argument holds for $\lambda + \xi$ positive or negative alike.

[9] Indeed, suppose leader 1 plays FF; if 2 plays CF then the common leader will play FF and get zero in the resulting FF outcome, since her alternative is to play CC to implement CF in which case she would get $(\lambda + \xi - Q)/2 < 0$; hence leader 1 will get zero. If on the other hand 2 does not play CF then the common leader will play CC and implement cooperation, where leader 1 gets 1.

time when the group leaders are locked in, and in those cases it makes sense for the common leader to be modeled as moving last.

The successful mediation of President Roosevelt in the Russo-Japanese War seems to fit the last picture reasonably well. He intervened after the war had been going on for more than a year, and his strong interest in maintaining good relations with both parties might suggest that the necessary incentives condition was satisfied.

However, cases of individual interventions in international affairs are not common. Especially in contexts of civil wars within nations the obvious candidate to play the role of common leader is the United Nations. To what extent can the factions involved in local conflicts punish the UN? Q stands for reputational loss, but since the scale of the regional conflicts where the UN operates is usually small it seems reasonable to presume that such loss is not so severe; in other words it seems appropriate to assume that Q here is essentially zero. The incentives of the common leader are then entirely determined by the parameters λ and ξ of the underlying game, and as we know they are conducive to cooperation in games where $\lambda + \xi < 0$. This condition may or may not be satisfied in particular cases. If it is not, remember that the probability of cooperation increases with the level of P, that is with the strength of the pressure groups can exert on their leaders.

Doyle and Sambanis (2000) and Doyle and Sambanis (2006) conduct a detailed analysis of the UN operations since its onset. They list 121 civil wars between 1945 and 1999; of these, 99 ended with a military victory or a truce, that is without successful third parties interventions; of the remaining 22, 14 ended with a negotiated settlement mediated by the UN, and in 12 of these cases there was no recurrence within 2 years from the settlement. So the UN was successful when it was able to advance a cooperative plan; but the prevalent outcome was conflict. These events could be interpreted as outcomes of conflicts without a common leaders, or with group leaders superseding the common leader's efforts, with low levels of punishments for the group leaders.[10] And in this cases this is what the model predicts: high frequency of conflict, and sporadic occurrences of the cooperative outcome proposed by the common leader.

In the context of internal politics the typical situation is different: conflicting political parties are locked in stalemate or have provoked some sort of crisis, and they resort to asking an external figure for help. In this cases also Q may stand for reputational loss, and it can be quite relevant.

A case in point is the 1952 US Presidential election. It was a contest between two very distant group leaders, and Dwight Eisenhower was asked to be a candidate by both parties.[11] Prior to the election he had no political affiliation, and a failed presidency would tarnish his reputation as a

[10] Indeed in the case of civil wars one may think that the punishment, seen as cost of failure, is particularly high (in the limit, death); but in war life is at risk whether you have promised victory or not, so that the additional punishment inflicted by followers is actually small.

[11] The Republican Robert A. Taft was an arch-conservative who opposed the New Deal, opposed US entry into World War II, opposed the Nuremberg trials, opposed NATO, the UN, and labor unions. The Democrat, Adlai Stevenson, along with Eleanor Roosevelt was the leader of the progressive movement: for labor unions and for expanding the New Deal, and a leading internationalist who was instrumental in the founding of the UN. See Cotter (1983).

war hero. In office Eisenhower did act for the public good, and his success at bringing the nation together is indicated by the fact that he has been the most popular president of the US post-war history.

5. Analysis of the Games

We first summarize the formal results; details are offered in the next three subsections.

Summary of Results. (1) In the game where the common leader moves after the group leaders: If $Q > \lambda+\xi$, the only equilibrium outcome is cooperation. If $Q < \lambda+\xi$ the only equilibrium outcomes are FC and CF if $P > -\xi$ and FF if $P < -\xi$.

(2) In the simultaneous moves game: If $P < -\xi$ the only equilibrium outcome is FF. If $P > -\xi$ the unique equilibrium outcome is a mixture of CC and FF, where the probability of CC goes to 1 as $P \to \infty$.

(3) In the game where the group leaders move after the common leader: Both for $P < -\xi$ and $P > -\xi$ there are equilibria as in the previous case; but equilibria with outcome FF also exist for any P.

We now turn to the detailed study of the equilibria in the three games. We study the three extensive form games defined in Section 2 in the following order: (1) the group leaders move first, then the common leader; (2) all leaders move simultaneously; and finally (3) the common leader moves first, then the group leaders.

5.1. The Group Leaders Move First, Then the Common Leader

We start with the sequential game where the group leaders simultaneously move first, and the common leader moves after them. The strategy set of each group leader is the set A of action profiles in the underlying game (her possible proposals). The common leader has 16 information sets - one for each of the possible 4×4 choices of the group leaders - and at each one she can choose a proposal in the set A; thus the strategy set of the common leader is the 16-fold Cartesian product of A. Incidentally, in the following proposition no weak dominance restriction is required: the results hold for *all* subgame perfect equilibria.

Proposition 2. *If the common leader moves after the group leaders:*

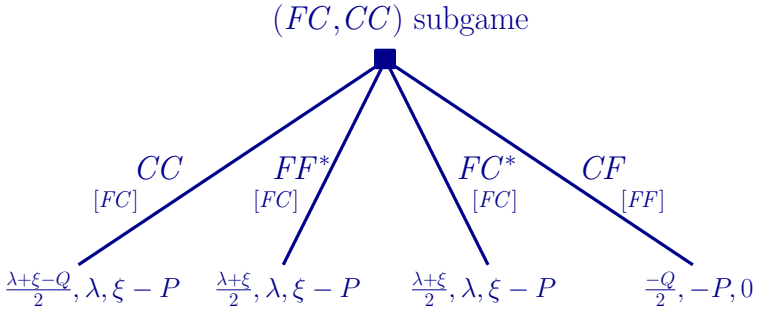
If $Q > \lambda + \xi$ the only equilibrium outcome is cooperation, CC;

If $Q < \lambda + \xi$ the only equilibrium outcomes are FC and CF if $P > -\xi$ and FF if $P < -\xi$.

Proof. Since the action set of each leader is $\{FC, FF, CC, CF\}$ there are 16 possible subgames corresponding to each pair of group leaders choices (and in each subgame the common leader has 4 choices). In each subgame, given the common leader's optimal choice the implemented action is determined, and this in turn determines the group leaders' payoffs. These payoffs can therefore be displayed in a 4×4 matrix (as in Table 1 below), the row and column players being respectively $\ell = 1$ and $\ell = 2$; each entry contains their payoffs in the corresponding subgame.

Start with the case $0 < \lambda + \xi < Q$. In Figure 1 we illustrate the working of the model in the subgame corresponding to the profile (FC,CC) of the two group leaders. The other cases are similar. If the common leader plays CC then the implemented action is FC, and the second group punish the two leaders they follow, namely their group leader and the common leader (who have promised 1 against a realized group payoff of ξ). Whence the leftmost payoff $(\lambda+\xi-Q)/2, \lambda, \xi - P$. The other payoffs are obtained similarly. Obviously the best replies of the common leader are FF and FC - by which she induces the FC outcome which gives her $(\lambda + \xi)/2$. Note that both choices yield the same payoffs to the group leaders. This latter fact is always true when the common leader has multiple best responses.

Figure 1: Shown is the (FC,CC)-subgame. Best responses of the common leader are stared. In brackets the implemented profile is shown.



Observe that whenever the common leader can induce the CC outcome without being punished she will do it because CC is her most preferred outcome. For each pair of group leaders actions the common leader best response(s) determine their payoffs; in the case just seen for example the group leaders payoff is $\lambda, \xi - P$. Table 1 displays the 16 possibilities (recall that we are assuming $Q > \lambda + \xi$). The corresponding common leader best responses are in brackets. The stars indicate the two players best responses. The unique pure equilibrium of the game clearly has both group leaders playing FF and the common leader playing CC.

Table 1: Group leaders' payoffs in the 16 subgames, with common leader's best responses in square brackets. For example the (FC,CC) entry $\lambda, \xi - P$ comes from the (FC,CC)-subgame shown in Figure 1, where the common leader's best responses are FF and FC.

	CF		FF		CC		FC	
FC	$-P, -P$	$[CC, FF]$	$-P, 0^*$	$[FF]$	$\lambda^*, \xi - P$	$[FF, FC]$	λ^*, ξ	$[FC]$
FF	$0^*, -P$	$[FF]$	$\mathbf{1}^*, \mathbf{1}^*$	$[CC]$	$1, 1^*$	$[CC]$	$1, 1^*$	$[CC]$
CC	$\xi - P, \lambda^*$	$[FF, CF]$	$1^*, 1$	$[CC]$	$1, 1$	$[CC]$	$1, 1$	$[CC]$
CF	ξ, λ^*	$[CF]$	$1^*, 1$	$[CC]$	$1, 1$	$[CC]$	$1, 1$	$[CC]$

Consider next the case $\lambda + \xi < 0$. Accept for a moment that there is no equilibrium where group leader 1 plays FC or leader 2 plays CF. Barring those plays the 4×4 payoff matrix becomes 3×3, and it is immediate to verify that in all cases the best response of the common leader is CC and cooperation obtains. It remains to exclude aggressive play by the group leaders in equilibrium. Suppose 1 plays FC. If 2 plays FC, FF or CF then since $\lambda + \xi < 0$ clearly the common leader plays

FF, implemented profile is FF and 1 gets $-P$. If 2 plays CC the common leader's best response now depends on Q (see Figure 1): if $Q < |\lambda + \xi|$ she will play CF (getting $-Q/2$ in the resulting outcome FF) and 1 will get $-P$; if $Q > |\lambda + \xi|$ the common leader will play FF or FC (getting $(\lambda + \xi)/2$ in the outcome FC), and 2 will get $\xi - P$. In both cases a group leader would get less than zero. But any group leader can guarantee herself zero by playing FF. Indeed, considering leader 1, if 2 plays CF the common leader will play FF, outcome will be FF and 1 will get zero; if 2 plays otherwise the common leaders will play CC and 1 will get 1. Obviously the argument holds for both group leaders.

Lastly, suppose $0 < Q < \lambda + \xi$. Assume $P > -\xi$. Low punishment $Q < \lambda + \xi$ for the common leader implies that she is better off at FC and CF, even if punished by one group, than at FF. Table 1 now becomes Table 2.

Table 2: Case $Q < \lambda + \xi$ and $P > -\xi$

	CF		FF		CC		FC	
FC	$-P, -P$	$[CC, FF]$	λ^*, ξ^*	$[CC]$	$\lambda^*, \xi - P$	$[FF, FC]$	λ^*, ξ^*	$[FC]$
FF	ξ^*, λ^*	$[CC]$	$1, 1$	$[CC]$	$1, 1$	$[CC]$	$1, 1$	$[CC]$
CC	$\xi - P, \lambda^*$	$[FF, CF]$	$1, 1$	$[CC]$	$1, 1$	$[CC]$	$1, 1$	$[CC]$
CF	ξ^*, λ^*	$[CF]$	$1, 1$	$[CC]$	$1, 1$	$[CC]$	$1, 1$	$[CC]$

All pure strategy equilibria of the leaders game have now outcomes FC or CF: either all play FC or all play CF and no leader gets punished; or the two group leaders play (FC, FF) or (FF, CF) and the common leader plays CC and gets punished.

To finish consider $P < -\xi$. The matrix is the same as in the case $P > -\xi$, what change are the group leaders' best responses: if 1 plays FC then 2's best response is CF, and vice versa. In the only equilibrium both group leaders play aggressively and get punished, and the common leader opts out by playing FF or CC. □

5.2. All Leaders move simultaneously

Of course here the only subgame is the whole game, so we just have to find the Nash equilibria in weakly undominated strategies. And recall that with simultaneous moves the strategy set of all leaders is the set A of profiles of the underlying game, hence the game is $4 \times 4 \times 4$. This case is a little more involved to analyze because mixed equilibria naturally arise.

Elimination of weakly dominated strategies considerably simplifies this game. Indeed, for a group leader $\ell = k \in \{1, 2\}$, a proposal $s^\ell = (a_1, a_2)$ is weakly undominated if and only if $a_\ell = F$. So leader $\ell = 1$ will only play FC or FF and $\ell = 2$ will only play CF or FF. This is proved in Lemma 1 in Appendix B. Given this, for the common leader the strategies CF and FC are (strictly) dominated by FF, so the common leader will only play CC or FF. This is Lemma 2 in Appendix B. Therefore the analysis is reduced to the $2 \times 2 \times 2$ game presented in Table 3, where the three payoffs in each entry are as usual ordered with the leaders' index (first common then the other two).

Table 3: The reduced game. The left panel shows utilities when the common leader plays CC; in the right panel are the payoffs when the common leader plays FF.

CC	CF	FF
FC	$0, -P, -P$	$\frac{\lambda+\xi-Q}{2}, \lambda, \xi$
FF	$\frac{\lambda+\xi-Q}{2}, \xi, \lambda$	$1, 1, 1$

FF	CF	FF
FC	$0, -P, -P$	$0, -P, 0$
FF	$0, 0, -P$	$0, 0, 0$

In the reduced game a strategy profile may be written as a vector of the form (q, p_1, p_2), q being the probability that the common leader plays CC, p_1 the probability that leader $\ell = 1$ plays FC and p_2 the probability that $\ell = 2$ plays CF.

The next result characterizes the equilibria of the leaders' game. All the equilibria are at least partially mixed, and the mixing probabilities are given in the next two displayed equations. Equation (4) below describes the profile where the common leader plays CC for sure and $p_1 = p_2 = \tilde{p}$:

$$\tilde{q} = 1\,, \qquad \tilde{p} \equiv \frac{\lambda - 1}{\lambda - 1 + P + \xi}. \tag{4}$$

Note that $\tilde{p}$ converges to 0 as P becomes large so the implemented outcome converges to cooperation as P becomes large. The pair $(\hat{q}, \hat{p})$ in (5) below describes a fully mixed profile, where $p_1 = p_2 = \hat{p}$.

$$\hat{q} \equiv \frac{P}{P + \lambda - 1 - \hat{p}(P + \lambda + \xi - 1)}\,, \qquad \hat{p} \equiv \frac{1}{1 + Q - (\lambda + \xi)} \tag{5}$$

In Appendix B it is proved that the equilibria are the following:

Proposition 3. *In the simultaneous moves game:*
If $P < -\xi$ the equilibria are all $(q, 1, 1)$ for $-P/\xi < q \leq 1$, with outcome FF.
If $P > -\xi$:
If $P + \xi < (\lambda - 1)(Q - (\lambda + \xi))$ the unique equilibrium is $(\hat{q}, \hat{p}, \hat{p})$
If $P + \xi > (\lambda - 1)(Q - (\lambda + \xi))$ the unique equilibrium is $(1, \tilde{p}, \tilde{p})$.

In this game, if P is small ($P < -\xi$) then the equilibrium outcome is FF, just as in the underlying game; for fixed Q, if P is large then the relevant equilibrium is $(1, \tilde{p}, \tilde{p})$: the common leader plays CC for sure, the probability of cooperation increases with P, and as $P \to \infty$ the probability of aggressive play by the group leaders is small for obvious reasons and in fact it goes to zero, so the cooperative outcome obtains with probability 1 in the limit. This equilibrium is independent of Q.

Large Q alone is not sufficient for cooperation with high probability. The relevant equilibrium becomes $(\hat{q}, \hat{p}, \hat{p})$, and if $Q \to \infty$ then the probability of cooperation goes to 1 only if also $P \to \infty$. On the other hand, in this equilibrium when $P + \xi$ and $Q - (\lambda + \xi)$ are close to zero fighting emerges with probability close to 1.

5.3. The Common Leader Moves First, Then the Group Leaders

We lastly consider the sequential game where the common leader moves first. In this case a group leader has four information sets, each corresponding to a proposal chosen by the common

leader; her strategy set is thus the 4-fold Cartesian product of A. We shall see that in this case, unlike in the simultaneous moves version, there are equilibria where the outcome is FF for all values of P; in all of them all leaders are worse off than in the corresponding equilibria of the simultaneous moves game.

The weak dominance arguments in the reduction lemmas 1 and 2 still apply to this extensive form. For the sake of completeness the relative statements appear as Lemmas 9 and 10 in Appendix C. As before, the proposals CF and FC of the common leader are dominated, therefore the group leaders have only two relevant information sets (corresponding to choices CC and FF of the common leader); and since their undominated proposals are only FC and FF for leader 1 and CF and FF for leader 2 the game is that of Table 3, reproduced here for convenience.

Table 3: The reduced game. The left panel shows utilities when the common leader plays CC; in the right panel are the payoffs when the common leader plays FF.

CC	CF	FF
FC	$0, -P, -P$	$\frac{\lambda+\xi-Q}{2}, \lambda, \xi$
FF	$\frac{\lambda+\xi-Q}{2}, \xi, \lambda$	$1, 1, 1$

FF	CF	FF
FC	$0, -P, -P$	$0, -P, 0$
FF	$0, 0, -P$	$0, 0, 0$

By backward induction we restrict attention to Nash equilibria in each subgame, where each such equilibrium is a pair of proposals, one by each group leader. It is easiest to look at these equilibria right away. The FF-subgame has the unique equilibrium $\phi \equiv (FF, FF)$, with implemented profile FF, for any value of P. Equilibria in the CC-subgame depend on the value of P.

If $P < -\xi$ then in the CC-subgame between the group leaders the unique equilibrium is the aggressive play $\alpha \equiv (FC, CF)$, with implemented profile FF. Therefore the only equilibrium pair in the two subgames is (α, ϕ) - α in the CC-subgame and ϕ in the FF-subgame. If $P > -\xi$ the CC-subgame has three equilibria: two in pure strategies, $\eta^1 \equiv (FC, FF)$ and $\eta^2 \equiv (FF, CF)$ with outcomes respectively FC and CF; and a mixed equilibrium with

$$p_1 = p_2 = \frac{\lambda - 1}{\lambda - 1 + P + \xi} \equiv \tilde{p}$$

(where p_1 is the probability that 1 plays FC, p_2 the probability that 2 plays CF). Possible equilibrium pairs are then $(\eta^1, \phi), (\eta^2, \phi)$ and $(\tilde{p}, \phi)$.

Equilibria of the leaders game where the common leader uses a pure strategy will be written as for example $(FF, (\eta^1, \phi))$ - meaning that the common leader plays FF and the equilibria played in the two subgames are respectively η^1 and ϕ. Recall that in the equilibrium $(CC, (\tilde{p}, \phi))$ the probability of cooperation depends on P, and it goes to 1 as $P \to \infty$. We are now ready to state

Proposition 4. *In the sequential game where the common leader moves first:*

If $P < -\xi$ the only equilibrium outcome is FF; the equilibria are $(CC, (\alpha, \phi))$, $(FF, (\alpha, \phi))$ and those where the common leader mixes between CC and FF.

If $P > -\xi$ and $Q < \lambda + \xi$ the profile $(CC, (\tilde{p}, \phi))$ is an equilibrium; the other equilibria are $(CC, (\eta^1, \phi)), (CC, (\eta^2, \phi))$ with outcomes FC and CF respectively.

If $P > -\xi$ and $Q > \lambda + \xi$ the equilibria $(FF, (\eta^1, \phi))$ and $(FF, (\eta^2, \phi))$ with outcome FF exist, together with another equilibrium: $(FF, (\tilde{p}, \phi))$ also with outcome FF if $P < -\xi + (\lambda - 1)(Q - (\lambda + \xi))$; and $(CC, (\tilde{p}, \phi))$ if $P > -\xi + (\lambda - 1)(Q - (\lambda + \xi))$.

Proof. If $P < -\xi$ the only equilibrium pair in the subgames is (α, ϕ), and in either of them the common leader gets zero; so she is indifferent between CC and FF.

Suppose now $P > -\xi$. Assume first $0 < Q < \lambda + \xi$. If the continuation equilibria are (η^1, ϕ) and (η^2, ϕ) obviously the common leader prefers CC to FF. Given continuation $(\tilde{p}, \phi))$ the same holds if and only if the CC payoff, which is $(1-p)^2 + 2p(1-p)\frac{\lambda+\xi-Q}{2}$, is larger than zero at $\tilde{p}$; this occurs if and only if $P > -\xi + (\lambda - 1)(Q - (\lambda + \xi))$, which is implied by $0 < Q < \lambda + \xi$.

Next consider $\lambda + \xi < 0$ or $0 < \lambda + \xi < Q$. If in the CC-subgame the group leaders play η^1 or η^2 then by playing CC the common leader gets $\frac{\lambda+\xi-Q}{2} < 0$ (punished by the group whose leader plays FF), so she will play FF. If on the other hand the continuation is $(\tilde{p}, \phi))$ the conclusion follows from the analysis of the previous case. □

Here the cooperative outcome CC may emerge only in some of the many equilibria, mainly if P is large. Compared to the simultaneous moves version, here there are two new equilibria with outcome FF for however large $P > -\xi$. Consider $\eta^1 \equiv (FC, FF)$ for illustration. The point is that if leader 1 plays FC and the common leader plays CC leader 2 is better off playing FF getting ξ than fighting back with CF which yields $-P$. But at (FC, FF) the common leader playing CC would get punished by group 2 and get a negative payoff; her best response is FF which guarantees her zero. So if one of the group leaders intends to play aggressively a cooperative proposal by the common leader is not viable, and all leaders end up playing FF.

Overall, having the common leader move first is less favorable to cooperation since in htis case there are unfavorable equilibria that do not exist in the simultaneous move case.

6. Conclusions

We have studied how political leadership can fundamentally alter outcomes in group conflicts. We rely on a model of leadership which may be a useful model in general environments. Given an underlying game among groups, we construct a game among leaders in which the leaders' strategies are action profiles proposed by each leader to the groups, which can be interpreted as "recommendations and promises". Groups choose among the proposals to maximize their utility. Our particular attention, differently from existing literature on leadership, is on the role of "common leaders" acting as *super partes* mediators, who have regard for common interests to all groups and try to maximize aggregate welfare.

The main insight derived from the analysis of the model is that conflict in polarized societies can be substantially reduced thanks to the intervention of such figures. A common leader will induce cooperation if she steps in after the group leaders have advanced their proposals, provided she is accountable; and this is the case if the punishment she may have to suffer is higher than her payoff from a winner/loser outcome.

In games where the above conditions fail the best equilibrium outcome is cooperation with high probability, which requires that the value of punishment to the group leaders be sufficiently large.

Our setup relies on simplifying assumptions, some of which may be in contrast with important real world regularities. In the model, leaders share precisely the utility of their constituencies, so their incentives are perfectly in line with those of the groups. In particular, leaders do not have a political career to pursue, nor derive utility from being leaders; and they cannot profit directly or indirectly on their position. Followers, on their part, make the task of the leaders as easy as possible: they hear what the leaders say, and take their promises at face value, with the understanding that punishment will follow if the leader does not deliver. Finally, punishments matter and in the model they are just numbers; specific research would be needed to gauge the realistic ranges of the losses groups can impose on leaders in concrete situations.

Appendix A. Proof of Proposition 1

This result is in fact true for any leaders game, with any number of groups. Observe that the model trivially extends to the case of K groups: just take $k, \ell \in \{1, 2, \ldots, K\}$ instead of $k, \ell \in \{1, 2\}$. Proving the statement for this more general case requires no additional effort, so we state it for this case:

Statement. For any leaders game, if each group only considers the proposal of their own group leader, then at the Nash equilibria of the leaders game the distributions of action profiles chosen by the groups are the same as those induced by the Nash equilibria of the corresponding underlying game.

Proof. For a mixed strategy $\hat{\sigma}^k$ of leader k we let $\hat{\sigma}^k_{A_k}$ the induced distribution on A_k. Our first claim is that

$$\forall \hat{\alpha} \in NE(UG) \exists \hat{\sigma} \in NE(LG) : \forall k,\ \hat{\sigma}^k_{A_k} = \hat{\alpha}_k, \tag{A.1}$$

where $NE(UG)$ and $NE(LG)$ denote the sets of Nash equilibria of the underlying game and leaders' game respectively. Consider a mixed action profile $\hat{\alpha} \in NE(UG)$. For any action $b_k \in \text{supp}(\hat{\alpha}_k)$ choose

$$a_{-k}(b_k) \in \text{argmin}_{c_{-k} \in A_{-k}} u_k(b_k, c_{-k}). \tag{A.2}$$

Define now $\hat{\sigma}^k$ as:

$$\hat{\sigma}^k(a) \equiv \sum\nolimits_{a_k \in A_k} \hat{\alpha}(a_k) \delta_{(a_k, a_{-k}(b_k))}(a). \tag{A.3}$$

If all leaders j different form k follow the strategy defined in (A.3) then leader k is facing the probability on A^{-k} given by $\hat{\alpha}_{-k}$. Consider now a possible strictly profitable deviation $\hat{\tau}^k$ from $\hat{\sigma}^k$. Since by following $\hat{\sigma}^k$ the k leader incurs no punishment cost, the increase in net utility to leader k from $\hat{\tau}^k$ is at least as large as the increase in direct utility, and the direct utility is the utility of the followers. Thus $\hat{\tau}^k$ would have a marginal on A_k that is a profitable deviation for player k from $\hat{\alpha}_k$ against $\hat{\alpha}_{-k}$, a contradiction with $\hat{\alpha} \in NE(UG)$.

The second claim is:

$$\forall \hat{\sigma} \in NE(LG),\ \text{if } \hat{\alpha}_k \equiv \hat{\sigma}^k_{A_k},\ \text{then } \hat{\alpha} \in NE(UG). \tag{A.4}$$

Consider in fact a strictly profitable deviation β_k from $\hat{\alpha}_k$ of a player k in the underlying game. Extend β_k to a profitable deviation τ^k in the leaders game of the k^{th} group leader following the construction in equations (A.2) and (A.3). This deviation would insure for group leader k, the same utility as β_k, which would then be higher than $\hat{\sigma}^k$, since the direct utility of τ^k is higher than $\hat{\sigma}^k$, and its punishment cost is zero; a contradiction with the assumption that $\hat{\sigma}^k$ is a best response. □

Appendix B. Proofs for the Simultaneous Moves Game

Lemmas for the Reduction

Lemma 1. *For group leader $\ell = k \in \{1, 2\}$ the strategy s^k is weakly dominated if and only if $s^k_k = C$.*

Proof. We first show that strategies with $s^k_k = C$ are dominated by FF. Let $a, b \in A$ denote the strategies chosen by the two groups, and $g = (a_1, b_2)$ the implemented profile. Take $k = 1$.

Consider $s^1 = CF$ first. If $s^0 \in \{FF, FC\}$ then $g = (F, b_2)$ and direct utility is $U^1(g) \geq u_1(FF)$, and the same occurs if $s^1 = FF$; the inequality implies that there is no punishment either way, so under both strategies $V^1(g) = U^1(g)$. Suppose now $s^0 \in \{CF, CC\}$. If $s^0 = CF$ then $g = CF$ and $V^1(g) = \xi < 0$; on the other hand if $s^1 = FF$ then $g = FF$ so $V^1(g) = 0$ (no punishment since $u_1(g) = u_1(s^1)$). If $s^0 = CC$ then $g = (C, b_2)$ and $V^1(g) = u_1(g)$ (no punishment because leader 1 is not followed by her group); under FF nothing changes.

Consider now $s^1 = CC$. If $s^0 = FC$ then $g = (F, b_2)$ and $V^1(g) = u_1(g)$ (no punishment since leader 1 is not followed), and the same holds if $s^1 = FF$. If $s^0 = CF$ then $g = CF$ therefore $s^1 = CC$ yields $V^1(g) = \xi - P < 0$, while under $s^1 = FF$ we would have $g = FF$ and $V^1(g) = 0$ (no punishment since $u_1(g) = u_1(s^1)$). If $s^0 = CC$ then $g = (C, b_2)$ and $V^1(g) \leq u_1(g)$ (if $b_2 = F$ the inequality is strict because leader 1 is punished); in this case $s^1 = FF$ yields the unfollowed leader 1 payoff $V^1(g) = u_1(g)$. Suppose finally that $s^0 = FF$; if $b_2 = F$ then $g = CF$ and $V^1(g) = \xi - P$ while if $s^1 = FF$ then $g = FF$ and $V^1(g) = 0$ (no punishment since $u_1(s^1) = u_1(g)$); if $b_2 = C$ then $g = CC$ and $V^1(g) = u_1(CC) = 1$; but if $s^1 = FF$ then $g = FC$ whence $V^1(g) = u_1(g) = \lambda > 1$ (no punishment since $u_1(g) > u_1(s^1)$).

To show that any strategy with $s^k_k = F$ is not weakly dominated note that $s^1 = FF$ is a unique best response to $s^0 = CF$, and $s^1 = FC$ is a unique best response to $s^0 = s^2 = CC$. □

From Lemma 1 follows

Lemma 2. *After eliminating the dominated strategies in Lemma 1, strategies CF and FC for the common leader are strictly dominated.*

Proof. We do it for CF. This proposal is rejected by group 1 who will play F (because $s^1 \in \{FC, FF\}$), and accepted for sure by group 2; so the implemented profile is FF and group 2 will punish the common leader. She is better off by playing FF (which yields zero), strictly for any $Q > 0$. □

Proof of Proposition 3

We restate Proposition 3. The analysis is organized considering three possible cases for the value of q, namely $q = 0$, $q = 1$ and then $q \in (0, 1)$.

Statement. In the leaders game:

If $P < -\xi$ the equilibria are all $(q, 1, 1)$ for $-P/\xi < q \leq 1$, with outcome FF.

If $P > -\xi$:

If $Q < \lambda + \xi$ the equilibria are $(1, 1, 0)$ and $(1, 0, 1)$ with outcomes FC and CF, and $(1, \tilde{p}, \tilde{p})$
If $Q > \lambda + \xi$ and $P + \xi < (\lambda - 1)(Q - (\lambda + \xi))$ the unique equilibrium is $(\hat{q}, \hat{p}, \hat{p})$
If $Q > \lambda + \xi$ and and $P + \xi > (\lambda - 1)(Q - (\lambda + \xi))$ the unique equilibrium is $(1, \tilde{p}, \tilde{p})$.

Proof. There is no equilibrium with $q = 0$ for any $P > 0$, from Lemma 3. Consider $P < -\xi$. We have equilibrium $(1, 1, 1)$ from Lemma 4; from Lemma 7 we have $(q, 1, 1)$ for $\max\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\} < q < 1$; and from Lemma 8 the same equilibrium for $-\frac{P}{\xi} < q < \frac{P}{P+\lambda-1}$ if that interval is nonempty. The last two give $(q, 1, 1)$ for $-\frac{P}{\xi} < q < 1$. Therefore if $P < -\xi$ we have $(q, 1, 1)$ for $-\frac{P}{\xi} < q \leq 1$, as in the statement. Turn to $P > -\xi$. For $Q < \lambda + \xi$ Lemma 5 gives $(1, 0, 1)$, $(1, 1, 0)$ and $(1, \tilde{p}, \tilde{p})$; for $Q > \lambda + \xi$ Proposition 5 gives $(\hat{q}, \hat{p}, \hat{p})$ if $P + \xi < (\lambda - 1)(Q - (\lambda + \xi))$, and Lemma 5 gives $(1, \tilde{p}, \tilde{p})$ for the reverse inequality. □

Lemmas for Proposition 3

We concentrate on the interesting cases in which the relevant inequalities among combinations of parameters hold strictly.

Equilibria with $q = 0$. We start with the fact that there are no such equilibria.

Lemma 3. *If $P > 0$, there is no equilibrium with $q = 0$*

Proof. If the common leader sets $q = 0$ then the leaders' game is the right panel of table 3 (ignoring the common leader's utility). This game has a unique Nash Equilibrium in dominant strategies in which both group leaders play FF. At this profile of actions of group leaders, CC yields 1, and FF yields 0, to the common leader, hence setting $q = 1$ is the best response. □

Equilibria with $q = 1$. We deal in turn with small P and larger P:

Lemma 4. *If $P < -\xi$ then there is a unique equilibrium with $q = 1$, with $(q, p_1, p_2) = (1, 1, 1)$.*

Proof. Since $\lambda > 1$ and $\xi < -P$, if $q = 1$ we see from table 3 that the action FC is dominant for the first group leader CF for the second). When group leaders play the action profile (FC, CF) then both CC and FF give utility 0 to the common leader, hence $(1, 1, 1)$ is the only equilibrium with $q = 1$. □

Lemma 5. *If $P > -\xi$:*

1. *There are two equilibria where group leaders play pure strategies: $(q, p_1, p_2) \in \{(1, 0, 1), (1, 1, 0)\}$ if and only if $Q < \lambda + \xi$. In these equilibria the outcome is FC or CF.*
2. *There is an equilibrium where group leaders play a mixed strategy if and only if:*

$$P + \xi > (\lambda - 1)(Q - (\lambda + \xi)) \tag{B.1}$$

The mixed strategy is $\tilde{p}$ in equation (B.2).

Proof. If $P > -\xi$ then at $q = 1$ the game among group leaders has three equilibria, the two pure profiles (FF, CF), (FC, FF) and a mixed one with:

$$p_1 = p_2 = \frac{\lambda - 1}{\lambda - 1 + P + \xi} \equiv \tilde{p} \tag{B.2}$$

Note that $\lambda > 1$ and our assumption that $P > -\xi$ insure that $\tilde{p} \in (0, 1)$.

We first consider the possible equilibria where group leaders play pure strategies:

1. If $\lambda + \xi - Q > 0$ then there are two equilibria, $(q, p_1, p_2) = (1, 0, 1), (1, 1, 0)$. This follows because CC gives $(\lambda + \xi - Q)/2$, while FF gives 0 to the common leader.
2. If $\lambda + \xi - Q < 0$ then there are no equilibria $(1, p_1, p_2)$ with $p_i \in \{0, 1\}$, because in this case the utility to the common leader from CC is lower than the one from FF.

We then consider the the possible equilibria where group leaders play a mixed strategy. At any mixed strategy profile (p, p), with $p \in (0, 1)$ of the group leaders the common leader playing CC gets

$$(1 - p)^2 + 2p(1 - p)\frac{\lambda + \xi - Q}{2}$$

and at $\tilde{p}$ this is larger than 0 (hence CC better than FF) if and only if (B.1) holds. □

Equilibria with $q \in (0, 1)$. To set up the analysis we assume that the common leader is playing q and compare a group leader's payoffs from FC and FF for each of the two possible strategies CF and FF of the other group leader. From Table 3 we see that in the first case FC is better than FF if and only if

$$q > -P/\xi \tag{B.3}$$

while in the second case FC is better than FF if and only if

$$q > \frac{P}{P + \lambda - 1} \tag{B.4}$$

In lemmas 6 and 7 we consider the two extreme possible cases for q:

Lemma 6. *There is no equilibrium with* $0 < q < \min\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\}$.

Proof. The condition on q implies that the action FF is dominant for both group leaders, but the common leader's best response to (FF, FF) is $q = 1$. □

Lemma 7. *There is an equilibrium with any q such that* $\max\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\} < q < 1$*, of the form* $(q, 1, 1)$.

Of course the set of such q's may be empty; this is the case when $P > -\xi$.

Proof. The condition on q implies that (FC, CF) is dominant for the group leaders, and at this profile the common leader gets zero both from CC and FF; the conclusion follows. □

Next we consider the intermediate cases for the values of q. At these values of q the game between the group leaders has three equilibria, two pure strategies and one mixed. We deal with pure strategies of group leaders in lemma 8. Observe that $\frac{P}{P+\lambda-1} < -\frac{P}{\xi}$ iff $P > 1-(\lambda+\xi)$.

Lemma 8. 1. *If $\frac{P}{P+\lambda-1} < q < -\frac{P}{\xi}$ then there is no equilibrium with $p_i \in \{0,1\}$ (that is, with group leaders playing pure strategies)*

2. *For any value $-\frac{P}{\xi} < q < \frac{P}{P+\lambda-1}$, there is an equilibrium in pure strategies for group leaders of the form $(q,1,1)$.*

Proof. For the first case, the two pure strategy equilibria in the resulting group leaders game are (FF,CF) and (FC,FF); consider the first (the second is analogous). In this case CC gives $\frac{\lambda+\xi-Q}{2}$, and FF gives 0. Considering only the cases in which the inequalities holds strictly, it follows that the best response of the common leader to this strategy profile of the group leaders is either $q=0$ or $q=1$, hence not in the open interval $(0,1)$.

For the second case, with q in that range the two pure strategy equilibria in the group leaders' game are (FC,CF) and (FF,FF). At the first profile the common leader gets zero from either CC or FF whence the equilibria; at the second one the common leader gets 1 from CC and zero from FF, hence the best reply is not interior. □

We lastly deal with the case of fully mixed equilibrium.

Proposition 5. *An equilibrium with $\min\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\} < q < \min\{\max\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\}, 1\}$ exists, with the mixed strategy $(\hat{q},\hat{p},\hat{p})$ defined in equation (5), if and only if $Q > \lambda+\xi$ and $P+\xi < (\lambda-1)(Q-(\lambda+\xi))$.*

Proof. For q to be part of a fully mixed equilibrium the common leader has to be indifferent between CC and FF, which is true if and only if $(1-p)^2 + 2p(1-p)\frac{\lambda+\xi-Q}{2} = 0$ that is if

$$p = \frac{1}{1+Q-(\lambda+\xi)} \equiv \hat{p} \tag{B.5}$$

Note that $0 \leq \hat{p} \leq 1$ if and only if $Q \geq \lambda+\xi$. On the other hand the indifference for group leader 1 (for example) between FC and FF requires:

$$-pP + (1-p)(q\lambda - (1-q)P) = pq\xi + (1-p)q$$

which is rewritten as:

$$p = \frac{P+\lambda-1-P/q}{P+\lambda+\xi-1} \equiv f(q) \tag{B.6}$$

Combining equations B.5 and B.6 we conclude that an equilibrium with q in the range exists if $0 < q < 1$, $f(q) = \hat{p}$ and

$$\min\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\} < q < \max\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\}.$$

Observe that $\frac{P}{P+\lambda-1} < -\frac{P}{\xi}$ if and only if $P+\lambda+\xi-1>0$, in which case f is strictly increasing; and f is strictly decreasing if the inequalities are reversed. Since $f(\frac{P}{P+\lambda-1})=0$ and $f(-\frac{P}{\xi})=1$, there is unique $\hat{q}$ in the given range such that

$$f(\hat{q}) = \hat{p}. \tag{B.7}$$

it is easy to check that this $\hat{q}$ is indeed the value in equation (5).

If $\max\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\} < 1$ - that is if $P < -\xi$ - we are done. For $P > -\xi$ we have $\max\{-\frac{P}{\xi}, \frac{P}{P+\lambda-1}\} = -\frac{P}{\xi} > 1$, hence in this case we must check whether an equilibrium exists with $\frac{P}{P+\lambda-1} < q < 1$. Since $f(\frac{P}{P+\lambda-1})=0$ and in this case f is increasing, the equilibrium exists if $f(1) > \hat{p}$, that is if

$$\frac{\lambda-1}{P+\lambda+\xi-1} > \frac{1}{Q-(\lambda+\xi-1)}$$
$$P+\xi < (\lambda-1)(Q-(\lambda+\xi))$$

where we have used that (since $P > -\xi$) $P+\lambda+\xi-1>0$ and that $Q-(\lambda+\xi-1)>0$ because $\hat{p} \in (0,1)$. If the inequality is false then the conditions for the equilibrium $(1, \tilde{p}, \tilde{p})$ are met. □

Appendix C. Case Where the Common Leader Moves First

We prove here the reduction lemmas stated in the text. We are considering the extensive form game in which the common leader chooses a proposal in the set A. Differently from the simultaneous move game, we proceed to the elimination of the strategies of the group leaders in each subgame induced by the choice of action of the common leader.

Lemma 9. *For any proposal by the common leader, for group leader $\ell = k \in \{1,2\}$ the proposal s^k is weakly dominated in the corresponding subgame if and only if $s^k_k = C$.*

Proof. The argument consists in showing that FF is always at least as good. The step is similar to the one given in the simultaneous move case. We spell it out for the CC-subgame.

As before let $a, b \in A$ denote the strategies chosen by the two groups, and $g = (a_1, b_2)$ the implemented profile, and take $k = 1$.

If leader 1 recommends C to her group when the common leader plays CC then $g = (C, b_2)$, whence $V^1(g) = u_1(g)$ (no punishment because leader 1 is not followed by her group); but under FF the implemented action and hence her payoff do not change. The other cases are analogous.

The proof for the other subgames is similar, and follows the pattern we have seen for the simultaneous move game.

To show that any strategy with $s^k_k = F$ is not weakly dominated note that FF is a unique best response in the CF-subgame, and FC is a unique best response in the CC-subgame if also leader 2 proposes CC. □

From Lemma 9 follows

Lemma 10. *After eliminating the dominated proposals in Lemma 9, proposals CF and FC for the common leader are strictly dominated.*

Proof. We do it for FC. Since in the FC-subgame leader 1 will play either FC or FF group 1 will play F. And however leader 2 plays in the subgame follower 2 will comply with te common leader recommendation; so the implemented profile will be FF and group 2 will punish the common leader who thus gets $-Q < 0$. She is strictly better off by playing FF which yields zero. □

Appendix D. Comparison with Correlated Equilibria

The leaders game built over an underlying game shares important features with the correlated equilibria of that underlying game: in both cases, thanks to a form of mediation, better outcomes than Nash equilibria can obtain; and in both solution concepts, leaders or the mediator suggest to followers an action profile, and followers respond. But the differences are deeper than the similarities.

In correlated equilibria the single mediator has no direct interest in the outcome; followers respond strategically to the action suggested privately to each, by updating the posterior on the action profile played by others, and would never want to punish the mediator. In the leaders game there are competing leaders with a direct interest in the outcome, so that their utility is affected by the action of the followers; the latter respond to the leaders' suggestions by choosing the best action profile from their point of view, and typically punish the chosen leaders with positive probability in equilibrium. Most importantly, although action profiles are implemented by the groups, the strategic interaction is among the leaders, not between the players of the underlying games.

We compare the sets of equilibrium action profiles taking as measurement of welfare the average utility of players in the underlying game (ignoring the welfare of the leaders which may include punishments).

Proposition 6. *If $P < -\xi$ the leaders equilibrium payoff is the same as the correlated payoff; otherwise it is strictly higher.*

Proof. The correlated payoff is zero. In the case where the group leaders move first we the group average payoff is 1. Consider next the simultaneous moves game. For $P < -\xi$ the leaders equilibrium payoff is zero. Turn to $P > -\xi$. The condition for average payoff in $(1, \tilde{p}, \tilde{p})$ to be positive is $\tilde{p}^2 * 0 + \tilde{p}(1-\tilde{p})(\lambda+\xi) + (1-\tilde{p})^2 * 1 > 0$, equivalently $P + \xi > -(\lambda-1)(\lambda+\xi)$. But the $(1, \tilde{p}, \tilde{p})$ equilibrium obtains in the range $P + \xi > (\lambda-1)(Q-(\lambda+\xi))$ which implies the condition above. Consider lastly $(\hat{q}, \hat{p}, \hat{p})$. If the common leader plays FF the possible outcomes are FC and FF both with positive probability hence average payoff is positive. If the common leader plays CC then the condition becomes as above $\hat{p}(\lambda+\xi) + (1-\hat{p}) > 0$ which is $Q > 0$. In the case where the common leader moves first, in the only equilibria not corresponding to those of the simultaneous case both groups get zero. □

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