# Credit Market Failures and Policy 

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#### Abstract

In a simplified version of the Stiglitz and Weiss (1981) model of the credit market we characterize optimal policies to correct market failures. Widely applied policies, notably interest-rate subsidies and investment subsidies, are compared to the theoretical optimum.


## 1. Introduction

Credit markets are a prototypical example of markets where the presence of asymmetric information plays an important role: Would-be borrowers have superior information on the quality of their project, or on the nature of their commitment to its success; and lenders have to design contracts taking into account their lack of information on these crucial aspects. Well known is the seminal paper by Stiglitz and Weiss (1981) on the subject with its implications for the inefficiency of market equilibrium with imperfect information. Less well known is the fact that a paper by Ordover and Weiss (1981) appeared in the same journal at the same time (actually a month before), pointing out explicit forms of legal intervention which would induce improvements upon market equilibrium. Five years later Mankiw (1986) showed how improvements on market outcomes could be obtained by the government through transfers, in particular investment subsidies.

Strangely enough, very few other papers ask the natural question of what kind of policy interventions may remedy the market failures typically generated by asymmetric information. One is de Meza and Webb (1987), which argues that unlike in the original Stiglitz-Weiss paper, credit

[^0]market equilibriummay be characterized by overinvestment, and that an investment tax can be an effective remedy. And another is Innes (1991), which presents an in-depth analysis of a model in which banks can screen firms via variable loan size; besides a full characterization of possible equilibria, that paper also contains a discussion of various possible policy interventions.

Surprisingly, existing literature does not provide, to the best of our knowledge, an analysis of optimal policies. This paper is meant to be a first step towards filling this gap.

We consider an economy in which banks have market power and offer debt contracts to privately informed firms. We assume that the government does not have a better information than the banks, and that it cannot sidestep the banking sector by offering credit directly to firms. The set of feasible policies consists of transfers (to the banks and or to the firms) conditional on publicly available information. Among these policies, we fully characterize the optimal ones for various parameter configurations.

Our optimality results are then applied to evaluating two widely used credit policies, interest-rate and investment subsidies. The former acts in a market in which adverse selection locks the agents in an inefficient equilibrium with too high an interest rate, which low-risk-low-return firms cannot afford; and it has the form of a conditional transfer to the banks. In the same kind of economy, investment subsidy relies on the idea that since the problem with adverse selection from the lender's point of view is the limited liability of the borrower, lowering the amount which the lender has to finance alleviates the problem; the government offers a transfer to the firms to finance a fraction of the required investment, thus reducing the bank's burden in case of failure of the project. In the model presented here, the interest-rate policy is unambiguously better than investment subsidy, in the sense of having higher difference between benefits and costs. It is also shown to be optimal-i.e., maximizing net benefits-in the class of all policies which the government can implement without extracting the borrowers' private information. Loan guarantees are another important policy instruments, both in the EU and in the United States; in the United States they constitute a prominent instrument of public intervention in favor of small business, under the so called 7(a) loan program (see www.sba.gov). This policy is also shown to have optimality properties. On the other hand, simple collateral provision on the part of the Government turns out to be suboptimal.

The paper is organized as follows. After introducing the model in Section 2, we discuss assumptions and differences with respect to the literature in Section 3. The model is studied next: after calculating market equilibrium in Section 4, the main results of the paper are presented in Section 5 (and the Appendix), where we formulate and solve the problem of optimal policy design. The most common real world credit policies are then analyzed and compared to the theoretical optimum in Section 6. Section 7 concludes.

## 2. Model

We analyze a simple model based on Stiglitz and Weiss (1981). There are two possible projects, A and B; both require a monetary investment $I$; and both have two possible returns: project $X=\mathrm{A}, \mathrm{B}$ yields $x>0$ with probability $p_{x}$ and 0 with probability $1-p_{x} .{ }^{1}$ Project A has higher expected return than B: $a p_{a}>$ $b p_{b}$; but project B is better if things go well: $b>a$. A natural interpretation is that B saves on costs, hence it gives a higher return but with a smaller probability (of course $p_{b}<p_{a}$ is implied by $a p_{a}>b p_{b}$ and $b>a$ ). The better project, A has positive net value: $a p_{a}>I$; for project B we shall separately consider the cases $b p_{b} \geq I$ and $b p_{b}<I$; in any case it will be $a p_{b}<I$. The firm/entrepreneur has wealth $W \geq 0$. It is assumed that $W$ cannot be used directly as investment, therefore the firm needs outside financing; and the only possible sources of funds are a bank or the government. The latter share the same imperfect information about the firm. If positive, firm's returns are seizable; and if wealth $W$ is positive it can be used as collateral; but this limits the firm's liability-no jail or other punishments in case of default. All agents will be assumed to be risk neutral. There are two types of firms A and B, and firm of type $X=\mathrm{A}, \mathrm{B}$ has access only to project $X$; the probability that a firm is of type A is denoted by $\lambda$. Bank and Government do not observe firms' types.

A contract between the bank and a firm, if any, is proposed by the bank. Such a contract specifies the amount advanced by the bank, fixed at $I$, and a pair ( $R, C$ ) where $R$ is the (total) amount the firm has to repay if the return $x$ to its project is positive, and $C$ is the amount of collateral, so that if $x=0$ the bank will get $C$. Notice that we are restricting the bank to offer debt contracts. The reason is that to implement a debt contracts the bank need only observe whether the firm has failed or not (in the present model, since profits are seizable, the firm will repay if it can); and this is the only observability requirement we impose. If the firm's profits were observable by the bank, the latter could offer a share finance contract and attain the first best (see the discussion at the end of the analysis of market equilibrium).

Before going on to the asymmetric information markets let us observe that with full information, an optimal debt contract for project $X$ is $(x, 0)$ if $x p_{x} \geq I$, and the null contract otherwise. This leaves the firm with zero (expected) profits, and the bank with $\max \left\{x p_{x}-I, 0\right\}$.

## 3. Modeling Choices

A large applied literature on credit market failures has focused on working capital loans in informal credit markets (see, e.g., the survey in Ghosh, Mookherjee, and Ray 2000), focusing on peer monitoring and group lending. Here, we have in mind a situation in which a formal banking system is

[^1]functioning and small-medium sized firms need capital investment to start up their business.

Our main structural assumptions are motivated by three stylized facts supported by some evidence in less favored regions of advanced economies: (1) there is abundance of deposits; (2) financing of fixed investment is more important than financing working capital; and (3) banks have strong market power. ${ }^{2}$ We next comment on how these facts relate to our modeling choices.

The meaning of "risky." Throughout the paper the riskier project B is a mean reducing (or preserving) spread of project $A$, as in the original paper by Stiglitz and Weiss. Alternatively one could take riskier to mean first order dominated (as in de Meza and Webb 1987, Innes 1991). With only two possible outcomes, this corresponds to assuming $a=b$. The model would then have different properties. In particular, when $W=0$, any contract $R$ accepted by type B would be accepted by type A too. The equilibrium that we study below, in which only riskier projects are financed, would not exist. Market failure in that model, if any, would mean that limited resources are "wasted" financing both types while a Pareto improvement could follow by focusing on type A. Given observation (1) above, this type of market failure seems to us less relevant that the one generated by the exclusion of safe projects due to high interest rates.

Monopolistic bank. In the original Stiglitz-Weiss paper and in most of the literature following it the banking sector is modeled as being competitive. For the economies we have in mind, the large observed interest spread (see footnote 2) suggests that, even in the presence of competition for deposits, the bank is in a strong bargaining position when facing a firm in need of funds.

Fixed loan size. A word is in order on our assumption that all firms demand a loan of identical size. If loan size were a choice variable for the firm, it could play a signaling role, as in Innes (1991). Indeed, even when $W=0$, the bank could easily separate types by proposing contracts of the form $(R, I)$. Loan size and interest payments are often linked in real world contracts, but it is hard to argue that full separation is attained. The variable loan model is useful to analyze the problem of allocating large projects to the best entrepreneurs. Our fixed loan model on the other hand is more adequate to analyze credit to a pool of firms with projects of roughly similar size; for a given loan size class there may still remain a significant problem of adverse selection.

[^2]
## 4. Market Equilibrium

Since collateral, if applicable, is paid by type $B$ with higher probability than by A, the bank may try to use it to separate types (by requiring collateral to type A only). If on the other hand $W=0$, then a contract only specifies repayment $R$, and no separation is possible (of course if two different $R$ 's are offered all types will select the lower). So the cases of null and positive initial wealth are studied separately.

We introduce the notation

$$
\zeta=b p_{b}-I
$$

the riskier project's expected value. We shall also use $\zeta^{+}$, defined as usual as $\max \{\zeta, 0\}$.

Case $W=0$. Here a contract is just a nonnegative number $R$. We define firm $X$ 's profit with contract $R$ and bank's profit on type $X$, respectively as ${ }^{3}$

$$
\begin{aligned}
u_{X}(R) & =p_{x}(x-R), \text { and } \\
v(R ; X) & =p_{x} R-I
\end{aligned}
$$

As observed above, by incentive compatibility the bank will propose, if any, a single contract; and given $R$, firm $X$ will accept the contract iff $p_{x}(x-$ $R) \geq 0$, i.e., iff $R \leq x$. Hence bank profits from contract $R$ are, letting $p_{\lambda}=$ $\lambda p_{a}+(1-\lambda) p_{b}$,

$$
v(R)= \begin{cases}p_{\lambda} R-I & \text { if } R \leq a  \tag{1}\\ (1-\lambda)\left(p_{b} R-I\right) & \text { if } a<R \leq b \\ 0 & \text { if } R>b\end{cases}
$$

and the bank will set $R$ equal to $a$ or $b$ in any contract; of course setting $R>b$ is equivalent to proposing no contract, and we shall use the latter terminology.

Suppose first $\zeta \geq 0$. The bank will set $R=a$ if

$$
\begin{equation*}
a p_{\lambda}-I \geq(1-\lambda)\left(b p_{b}-I\right) \tag{2}
\end{equation*}
$$

[^3]and $R=b$ otherwise; that is, respectively, if
$$
\lambda \gtreqless \frac{p_{b}(b-a)}{\left(p_{a}-p_{b}\right) a+b p_{b}-I} .
$$

Thus in equilibrium both types are financed if $\lambda$ is high enough. If the fraction of type-A firms (the good ones) is too small they will be left out of the market, and market equilibrium is inefficient.

The analysis is similar if $\zeta<0$. Here setting $R=b$ is out of the question, since on B-types the bank makes losses (the right hand side of Equation (2) is negative); it will then set $R=a$ if $a p_{\lambda}-I \geq 0$, and refuse to propose any contract otherwise. In this case the necessary and sufficient condition for the pooling equilibrium with all firms operating is therefore

$$
\lambda \geq \frac{I-a p_{b}}{a\left(p_{a}-p_{b}\right)}
$$

If contracts can only specify a repayment $R$, this equilibrium is (constrained) efficient when it exists. Indeed, any contract accepted by A-types will also be taken also by B-types, and for values of $\lambda$ satisfying the inequality above we have (from $a p_{\lambda} \geq I$ and $b>a$ ) that $\lambda a p_{a}+(1-\lambda) b p_{b}-I>0$ : it is better to have all firms financed than none.

We conclude that inefficient equilibrium-that is, A-types out of the market if $\zeta \geq 0$, and no contract if $\zeta<0$-occurs iff

$$
\begin{equation*}
\frac{I-b p_{b}+\zeta^{+}}{a p_{a}-b p_{b}} \leq \lambda<\frac{I-a p_{b}+\zeta^{+}}{a\left(p_{a}-p_{b}\right)+\zeta^{+}} \tag{3}
\end{equation*}
$$

If risky projects have nonnegative expected value, $\zeta \geq 0$, the first inequality is always satisfied. When $\zeta<0$ it reads $\lambda a p_{a}+(1-\lambda) b p_{b} \geq I$ : a minimal value of $\lambda$ is needed in order for the (constrained) efficient outcome to be both types operating. In both the cases, if $\lambda$ is not high enough, the market will be inefficient.

Case $W>0$. We assume in this case that $W \geq C^{\mathrm{A}}$, where $C^{\mathrm{A}}$, defined below, is the equilibrium collateral required by the bank to separate types. ${ }^{4}$

A contract is now a pair ( $R, C$ ), with firm $X$ 's profit and bank's profits on $X$ given, respectively by

$$
\begin{aligned}
& u_{X}(R, C)=W+p_{x}(x-R)-\left(1-p_{x}\right) C, \text { and } \\
& v(R, C ; X)=p_{x} R+\left(1-p_{x}\right) C-I .
\end{aligned}
$$

The bank can propose a pair of contract $\left(R^{X}, C^{X}\right), X=\mathrm{A}, \mathrm{B}$ trying to separate types.

[^4]Assume first $\zeta \geq 0$. Consider the pair $\left(R^{\mathrm{A}}, C^{\mathrm{A}}\right)$ and $(b, 0)$ with $R^{\mathrm{A}}, C^{\mathrm{A}}$ chosen so that the constraints on A's individual rationality and B's incentive compatibility are satisfied with equality:

$$
p_{a}\left(a-R^{\mathrm{A}}\right)-\left(1-p_{a}\right) C^{\mathrm{A}}=0, \quad 0=p_{b}\left(b-R^{\mathrm{A}}\right)-\left(1-p_{b}\right) C^{\mathrm{A}}
$$

This system has solution; letting $\gamma=p_{a}\left(1-p_{b}\right) / p_{b}\left(1-p_{a}\right)$ (larger than 1 since $p_{a}>p_{b}$ ), the solution is

$$
\begin{equation*}
R^{\mathrm{A}}=\frac{\gamma a-b}{\gamma-1}, \quad C^{\mathrm{A}}=\frac{p_{a} p_{b}(b-a)}{p_{a}-p_{b}} . \tag{4}
\end{equation*}
$$

Here $0<R^{\mathrm{A}}<a$ (the first being equivalent to $a p_{a} / b p_{b}>\left(1-p_{a}\right) /\left(1-p_{b}\right)$, true because the left hand side is $>1$ and the right hand side is $<1$; the second easily implied by A's IR constraint $a-R^{\mathrm{A}}=\left(1-p_{a}\right) C^{\mathrm{A}} / p_{a}$ and the value of $C^{\mathrm{A}}$ ) and $0<C^{\mathrm{A}}<R^{\mathrm{A}}$ (the first being evident, the second easily checked to be equivalent to $a p_{a}>b p_{b}$ which is true).

With this pair of contracts the bank separates types; and it is easy to check that the other IR and IC constraints are met. On the contract signed with A the bank makes

$$
\begin{aligned}
v\left(R^{\mathrm{A}}, C^{\mathrm{A}} ; A\right) & =p_{a} R^{\mathrm{A}}+\left(1-p_{a}\right) C^{\mathrm{A}}-I \\
& =p_{a} R^{\mathrm{A}}+p_{a}\left(a-R^{\mathrm{A}}\right)-I=a p_{a}-I,
\end{aligned}
$$

and on B's contract it makes $v(b, 0 ; B)=b p_{b}-I$. These are exactly the full information profits.

If $\zeta<0$, then the bank can propose just one contract, $\left(R^{\mathrm{A}}, C^{\mathrm{A}}\right)$; only A will apply (we are assuming that in case of indifference the firm does "what we want;" the contract can be modified so that incentives are strict), and the bank makes the same profits as before on type A, and nothing on type B. Again, the bank extracts all the gains from trade as if it had perfect information.

Therefore, if $W$ is high enough it works as a sorting device, and whatever the values of $\lambda$ and $\zeta$ the equilibrium outcome is efficient. Thanks to risk neutrality, the bank is able to separate the two types of firms without leaving any rent: contract ( $R^{\mathrm{A}}, C^{\mathrm{A}}$ ) is on the individual rationality line of both types.

Remark: Share finance. If the bank can observe $x$, the realization of the firm's return, it can finance the firm with a mix of debt and share finance. Then the equilibrium will be efficient even in the absence of collateral. Assume $W=0$ and let a contract be ( $\alpha, R$ ), where $\alpha$ is the fraction of the investment financed with debt. A fraction $(1-\alpha)$ of the investment $I$ is financed with a share contract, giving to the bank a right on a fraction $(1-\alpha)$ of the firms (postdebt) returns in case of success. Payoffs are:

$$
\begin{aligned}
u_{X}(\alpha, R) & =p_{x}[\alpha(x-R)], \text { and } \\
v(\alpha, R ; X) & =p_{x}[(1-\alpha)(x-R)+R]-I .
\end{aligned}
$$

If $\zeta \geq 0$, i.e., both types of projects have positive expected value, the bank can offer two contracts, $\left(\alpha^{\mathrm{A}}, R^{\mathrm{A}}\right)=(0,0)$ and $\left(\alpha^{\mathrm{B}}, R^{\mathrm{B}}\right)=$ $(1, b)$. Incentive compatibility and individual rationality constraints for the firms are satisfied, and the first best is achieved. If $\zeta<0$, again the first best can be achieved, this time by offering only the "pure share finance" contract $(0,0)$, taken only by the A-type firms. On the other hand, if $x$ is not observed by the bank, the B-type can always claim that $x=a$, and pay to the bank $(1-\alpha)(a-R)$. The nonobservability of returns thus destroys the incentive compatibility of the share finance contracts above.

## 5. Optimal Policies

### 5.1. Analysis

If firms have enough initial wealth, market equilibrium is efficient; we assume $W=0$. Then there is scope for policy only if condition (3) on p. 8 holds; we assume it does. Starting from an inefficient market equilibrium, the aim of policy is to act on agents' incentives to induce a more efficient outcome. Let us see what is involved before going to the details. If $\zeta \geq 0$, the inefficient market equilibrium has only B-types investing; since these projects have positive net worth, the problem is simply to have also A-types operating their projects, via a policy with minimal cost. If $\zeta<0$ on the other hand, inefficient market equilibrium has no firm financed; in this case "full" efficiency would be with only A-types operating; but this might be very costly to achieve, and it may happen that a policy inducing all firms to operate-a less efficient outcomeyields higher net benefits (think of $\zeta<0$ but small). The general problem, which we now formally address, is to consider both policies that induces pooling and policies that induce separation, and to compare their net gains.

In a setting with asymmetric information, the relevant notions of feasibility and efficiency are those that take into account the informational constraints. The first step is thus to define the class of policies which are compatible with the observability restrictions embodied in the model. Moreover, we assume that the Government (Gov) cannot directly intervene in the credit market, or force the bank to offer a specific type of contract. It takes the market power of the bank as given, but tries to influence the nature of the contracts offered through a policy of transfers.

A policy is a set of monetary transfers at dates 0 and 1 , conditional on observables. The latter are:

- at date 0 , actions:
- Bank proposes $\left(R^{\mathrm{A}}, C^{\mathrm{A}}\right),\left(R^{\mathrm{B}}, C^{\mathrm{B}}\right)$;
- Firm accepts $(R, C) \in \mathbb{R}^{2} \cup\{\emptyset\}$, where accepting $\emptyset$ means signing no contract;
- at date 1 , success/failure of the project. This we denote by $s: s=1$ [resp. 0] will mean success [resp. failure].

We shall use the following labels: $F$ for firm, $K$ for bank. A policy is then a pair of transfers $\left(t_{K}, t_{F}\right)$ :

$$
\begin{aligned}
t_{K} & =\left(t_{K, 0}\left(\left(R^{\mathrm{A}}, C^{\mathrm{A}}\right),\left(R^{\mathrm{B}}, C^{\mathrm{B}}\right)\right),\left(t_{K, 1}\left(\left(R^{\mathrm{A}}, C^{\mathrm{A}}\right),\left(R^{\mathrm{B}}, C^{\mathrm{B}}\right), s\right)\right)_{s=0,1}\right) \\
t_{F} & =\left(t_{F, 0}(R, C),\left(t_{F, 1}((R, C), s)\right)_{s=0,1}\right)
\end{aligned}
$$

Here $t_{K, 0}((\cdot),(\cdot))$ is the transfer to the bank if that contract is proposed; $t_{F, 0}(R, C)$ is the transfer to the firm if $(R, C)$ is accepted, etc. Notice that a transfer to one agent cannot be conditioned on another agent's action. For example, Gov cannot propose a transfer to firms conditional on the bank accepting a tax.

Given this set of feasible policies, the optimal policy problem is posed as follows. Before the intervention, the economy is at an inefficient equilibrium. Gov announces an incentive policy, $\left(t_{K}, t_{F}\right)$. Given the announced policy, the bank proposes new contracts that the firm may accept or refuse; if the firm refuses the bank's proposal, no contract is signed.

Welfare gains are obtained when the intervention induces the financing of additional projects with positive net value. The cost of each policy is measured by the sum of the transfers from Gov to the market participants. Our analysis consists of two parts: in the first we compute the minimal cost to induce a pooling equilibrium, in the second that of obtaining a separating equilibrium; comparing the net gains, the optimal policy is obtained for the various parameter configurations. The results of the analysis, reported in the Appendix, are summarized here.

PROPOSITION 1: The cost of an optimal pooling policy is $t_{*}=I-a p_{\lambda}+(1-$ $\lambda) \zeta^{+}$.

PROPOSITION 2: When $\zeta \geq 0$, the cost of an optimal separating policy is the same as the cost of an optimal pooling policy, $t_{*}$.

Starting from the inefficient equilibrium, a pooling policy creates total additional gains $\lambda\left(a p_{a}-I\right)+(1-\lambda) \min \left\{b p_{b}-I, 0\right\}$, while a separating policy creates additional gains $\lambda\left(a p_{a}-I\right)$. Thus optimal separation and pooling policies are equivalent in terms of net gains when $\zeta \geq 0$.

When $\zeta<0$, the cost of an optimal pooling policy is as in Proposition 1, while for separating policies we have

PROPOSITION 3: When $\zeta<0$, the cost of an optimal separating policy is $\frac{p_{a} p_{b}(b-a)}{p_{a}-p_{b}}-\lambda\left(a p_{a}-I\right)$.

When $\zeta<0$, comparison between separating and pooling policies must take into account that with separation the gains are larger than in the pooling
case ("lemons" are kept out of the market). The net gains of an optimal pooling policy are bigger than those of an optimal separating policy iff

$$
\frac{p_{a} p_{b}(b-a)}{p_{a}-p_{b}}-\lambda\left(a p_{a}-I\right)-(1-\lambda)\left(I-b p_{b}\right)>I-a p_{\lambda}
$$

which, after rearrangement, becomes

$$
p_{b}(b-a)\left(\frac{p_{a}}{p_{a}-p_{b}}-(1-\lambda)\right)>2(1-\lambda)\left(I-b p_{b}\right) .
$$

If $\zeta$ is close to zero, the inequality is satisfied and optimal policy is to induce pooling; for low values of $b p_{b}$, on the other hand, the opposite inequality may hold, and optimality would thus entail inducing separation. However, as we show in the Appendix, any optimal separating policy requires Gov to pay B-types outright to keep them away from the contract designed for A-types. This feature suggests that the implementation of separating policies may be problematic, because it presumes that Gov is able to discriminate firms from "nonfirms" on the basis of no action, i.e., of no evidence, else anyone would go and claim money from Gov.

### 5.2. Synthesis

We shall now draw a general picture of the results obtained.
Fix $a, b, p_{a}, I$ with $a<b, a p_{a}>I$. We can then parameterize economies by the riskiness of the "risky" projects, $p_{b} \in\left[0, \frac{a p_{a}}{b}\right]$, and the proportion of "safe" projects in the pool of firms, $\lambda$. A crucial threshold is whether $p_{b}$ is above or below $I / b$, that is whether $\zeta$ is above or below zero; in the latter case the pool of firms contains "lemons," projects that have a negative expected value.

For an optimal pooling policy, benefits will be larger than costs if the fraction $\lambda$ of good projects is not too low:

$$
\begin{equation*}
\lambda \geq \frac{2 \zeta^{+}+\left(I-a p_{b}\right)+\left(I-b p_{b}\right)}{2 \zeta^{+}+a\left(p_{a}-p_{b}\right)+a p_{a}-b p_{b}} \tag{5}
\end{equation*}
$$

Write (3) p. 8 as $\lambda_{1} \leq \lambda<\lambda_{3}$, and (5) as $\lambda \geq \lambda_{2}$; then it is elementarily checked that for any value of $p_{b}$ one has $\lambda_{1}<\lambda_{2}<\lambda_{3}$. For $\lambda_{1} \leq \lambda<\lambda_{2}$, market equilibrium is inefficient but using credit policy to force a pooling outcome is wasteful. For $\lambda_{2} \leq \lambda<\lambda_{3}$ on the other hand, a pooling policy is an effective remedy to market failure. The bounds $\lambda_{i}, i=1,2,3$ are functions of $p_{b}$, and the interval of values of $\lambda$ for which a pooling policy is effective shrinks as $p_{b}$ gets close to zero. The situation is summarized in Figure 1.

For $I / b \leq p_{b} \leq a p_{a} / b$ all projects generate positive expected return (the case considered in the Stiglitz-Weiss paper). For this parameter configurations $\lambda_{1}=0$, so financing all projects is always worthwhile. For intermediate values of $\lambda$ (in $\left[\lambda_{2}, \lambda_{3}\right]$ ) the optimal pooling policies discussed above generate higher benefits than costs. For lower values on the other hand neither


Figure 1: The general picture. Below $\lambda_{1}$ and above $\lambda_{3}$ market equilibrium is (constrained) efficient; in the shaded area, between $\lambda_{2}$ and $\lambda_{3}$, optimal pooling policies are desirable; between $\lambda_{1}$ and $\lambda_{2}$, they are wasteful, in that costs outweigh benefits.
pooling nor (as we know from the end of Section 5.1) separating policies have positive net benefits.

When $p_{b}<I / b$-the "lemon case"-the picture changes. Now $\lambda_{1}$ is positive: low values of $\lambda$ make the "no-contract" situation a better alternative than the pooling equilibrium. In this region of "very low quality environments"low $p_{b}$, low $\lambda$-the net gains of an optimal separation policy may be positive even though pooling policies are wasteful, the relevant inequality being

$$
\begin{equation*}
\lambda \geq \frac{p_{a} p_{b}(b-a)}{\left(a p_{a}-I\right)\left(p_{a}-p_{b}\right)} . \tag{6}
\end{equation*}
$$

### 5.3. Credit Policy versus Public Investment

Development policies may be seen as falling into the two categories of incentives to firms and provision of infrastructures, the latter including material infrastructures and human capital formation. For $\zeta \geq 0$, the model we present yields a simple insight on the issue of credit policy versus infrastructural investment: if the quality of the environment is "too low," then the costs of credit policy, even if optimally designed, outweigh its benefits. In these situations the real problem is not to foster investment but to improve the average quality of projects-that is, arguably, to raise $\lambda$ and/or $p_{b}$ through public investment. For $\zeta<0$ the same conclusion remains valid with the proviso that for some parameter range separating policies may be an alternative even if, as discussed in Section 5.1, they may be problematic to implement.

## 6. Credit Policy in Action

In our analysis of market equilibrium we saw that collateral may play an important role in facilitating the separation of firms' types. This suggests that a natural policy intervention may be direct provision of collateral to firms. Indeed, if Gov gives to all firms the minimal amount of collateral necessary to achieve separation, that is $C^{\mathrm{A}}=\frac{p_{a} p_{b}(b-a)}{p_{a}-p_{b}}$ (cf. Equation (4) on p. 9), market efficiency is restored. However, this policy is never optimal. Its cost for the Gov is $C^{\mathrm{A}}$; when $\zeta \geq 0$ this cost is larger than minimal cost $t_{*}\left(t_{*}=I-a p_{\lambda}+\right.$ $(1-\lambda) \zeta^{+}$is decreasing in $\lambda$, and takes value $p_{b}(b-a)<C^{\mathrm{A}}$ when $\left.\lambda=0\right)$; and when $\zeta<0$ suboptimality follows directly from Proposition 3.

The most commonly observed policies are of a different nature, in that they induce pooling. This should not be surprising given the suboptimality of direct collateral provision and the implementation problems which separating policies would face (cf. Section 5.1).

We now discuss the three policies mentioned in the introduction, namely interest rate subsidy, investment subsidy and loan guarantees. They are all pooling, hence generate the same expected gains with respect to the prepolicy equilibrium, $G=\lambda\left(a p_{a}-I\right)+(1-\lambda) \min \left\{b p_{b}-I, 0\right\}$; we compare their costs.

### 6.1. Interest Rate Subsidy

This policy takes the form of a conditional transfer to the bank: "If you charge $R \leq a$, I shall augment it to $(1+\beta) R$." Thus bank's profits in equation (1) p. 7 become $p_{\lambda} R(1+\beta)-I$ if $R \leq a$. Bank's prepolicy profits are $(1-\lambda) \zeta^{+}$. Gov chooses the least $\beta$ such that the bank prefers to finance all firms in the resulting equilibrium; therefore Gov chooses $\beta$ such that

$$
p_{\lambda}(1+\beta) a-I=(1-\lambda) \zeta^{+}
$$

that is,

$$
\begin{equation*}
\beta_{*}=\frac{I+(1-\lambda) \zeta^{+}}{a p_{\lambda}}-1 \tag{7}
\end{equation*}
$$

The (expected) cost of this policy is $p_{\lambda} \beta_{*} a=I-a p_{\lambda}+(1-\lambda) \zeta^{+} \equiv t_{*}$, which implies that it is an optimal policy. In terms of our notation, the policy is to set $t_{K}(R, C)=p_{\lambda} \beta_{*} a=t_{*}$ if $R \leq a$, and zero otherwise.

### 6.2. Investment Subsidy

Here Gov co-finances directly a share $(1-\theta)$ of the cost of any project, so that firms must only raise $\theta I$ on the market. As before, the optimal level of $\theta$ will be chosen by Gov so that at equilibrium all projects are financed. The bank's profits for given $\theta$ are:

$$
v(R)= \begin{cases}p_{\lambda} R-\theta I & \text { if } R \leq a  \tag{8}\\ (1-\lambda)\left(p_{b} R-\theta I\right) & \text { if } a<R \leq b \\ 0 & \text { if } R>b\end{cases}
$$

Since we start from an inefficient equilibrium, with $\theta=1$ either no project or only risky projects are financed. Gov induces financing of all firms by setting:

$$
\begin{equation*}
\theta_{*}=\frac{a p_{\lambda}-(1-\lambda) b p_{b}}{\lambda I} \tag{9}
\end{equation*}
$$

which is bigger than zero if $\lambda>\frac{p_{b}(b-a)}{\left(p_{a}-p_{b}\right) a+b p_{b}}$. Comparing with equation (3), we see that there is an interval of values of $\lambda$ for which $\theta_{*}>0$. The cost of this policy is $\left(1-\theta_{*}\right) I=\frac{I-a p_{\lambda}+(1-\lambda)\left(b p_{b}-I\right)}{\lambda}$, which is always bigger than $t_{*}$. This is evident for $\zeta \geq 0$; when $\zeta<0$ it follows from the condition that $\lambda>\frac{p_{b}(b-a)}{\left(p_{a}-p_{b}\right) a+b p_{b}}$. Investment subsidy is therefore not optimal.

### 6.3. Loan Guarantees

We now consider the loan guarantee policy, as for example in the SBA 7(a) program mentioned in the introduction. Gov guarantees a transfer $C$ to the bank in case of firm's default, conditional on the bank offering "cheap credit," $R \leq a$. Firm's payoff are unaffected, and bank's payoff becomes:

$$
v(R)= \begin{cases}p_{\lambda} R+\left(1-p_{\lambda}\right) C-I & \text { if } R \leq a  \tag{10}\\ (1-\lambda)\left(p_{b} R-I\right) & \text { if } a<R \leq b \\ 0 & \text { if } R>b\end{cases}
$$

The minimal value of $C$ that, starting from inefficiency, induces the bank to set $R=a$ and finance all firms is $C_{*}=\frac{I-a p_{\lambda}+(1-\lambda) \zeta^{+}}{1-p_{\lambda}}$. Again the expected cost of the policy is equal to $t_{*}$, which makes it optimal.

To conclude this section, a reinterpretation of the model in terms of standard monopoly theory may be useful. Our bank is a monopoly which, in the absence of collateral, cannot price discriminate. At the inefficient equilibrium, the bank produces a suboptimal quantity. Policies like an interest rate subsidy, or the loan guarantee just discussed, amount to a price subsidy that induces the bank to produce the optimal quantity, compensating it for the loss of surplus on the units it was already selling. As in the standard theory, this intervention brings efficiency.

## 7. Conclusion

This paper studies optimality properties of credit policy interventions designed to correct inefficiency of market equilibrium. It is shown that interest rate subsidy and loan guarantees are optimal, while investment subsidy and direct provision of collateral are not. If the quality of economic environment
is too low, then the costs of credit policy aimed at the individual firms outweigh its benefits, and public investment directed to improve the economy's potential may be more appropriate.

## Appendix

A policy is a pair of transfers $\left(t_{K}, t_{F}\right)$ :

$$
\begin{aligned}
t_{K} & =\left(t_{K, 0}\left(\left(R^{\mathrm{A}}, C^{\mathrm{A}}\right),\left(R^{\mathrm{B}}, C^{\mathrm{B}}\right)\right),\left(t_{K, 1}\left(\left(R^{\mathrm{A}}, C^{\mathrm{A}}\right),\left(R^{\mathrm{B}}, C^{\mathrm{B}}\right), s\right)\right)_{s=0,1}\right) \\
t_{F} & =\left(t_{F, 0}(R, C),\left(t_{F, 1}((R, C), s)\right)_{s=0,1}\right)
\end{aligned}
$$

where $t_{K, 0}((\cdot),(\cdot))$ is the transfer to the bank if that contract is proposed; $t_{F, 0}(R, C)$ is the transfer to the firm if $(R, C)$ is accepted, etc.

## A.1. Inducing a Pooling Equilibrium

Here Gov forces the bank to offer a single contract, and the bank will chose a contract $(R, C)$ so as to satisfy the participation constraints of both types. We can therefore simplify our notation:

$$
\begin{aligned}
& t_{0}=t_{F, 0}(R, C), \quad t_{1}(s)=t_{F, 1}(R, C, s) ; \quad t_{K 1}(s)=t_{K, 1}(R, C, s) ; \text { and } \\
& t_{X}^{F}=t_{0}+p_{x} t_{1}(1)+\left(1-p_{x}\right) t_{1}(0), t_{\lambda}^{F}=t_{0}+p_{\lambda} t_{1}(1)+\left(1-p_{\lambda}\right) t_{1}(0)
\end{aligned}
$$

also, let

$$
\begin{aligned}
t^{K}(R, C) & =t_{K, 0}(R, C)+p_{\lambda} t_{K 1}(R, C, 1)+\left(1-p_{\lambda}\right) t_{K 1}(R, C, 0) ; \text { and } \\
\mathcal{C} & =C \wedge\left(t_{0}+t_{1}(0)\right)
\end{aligned}
$$

where $\wedge$ is " $\min$ "- $t_{0}+t_{1}(0)$ is disposable income in $s=0$.
Given the announced policy, $(R, C)$ must be a best pooling contract for the bank, i.e., it must maximize the bank's profits subject to the individual rationality constraints; that is, it must solve

$$
\begin{aligned}
& \max _{R, C} p_{\lambda}\left(R+t_{K 0}(R, C)+t_{K 1}(R, C, 1)\right) \\
& \quad+\left(1-p_{\lambda}\right)\left(t_{K 0}(R, C)+t_{K 1}(R, C, 0)+\mathcal{C}\right)-I
\end{aligned}
$$

subject to

$$
p_{x}\left(x+t_{0}+t_{1}(1)-R\right)+\left(1-p_{x}\right)\left(t_{0}+t_{1}(0)-\mathcal{C}\right) \geq 0, \quad x=a, b
$$

These $\mathrm{IR}^{X}$ constraints may be written as

$$
p_{x} R+\left(1-p_{x}\right) \mathcal{C} \leq x p_{x}+t_{X}^{F}, \quad x=a, b
$$

also, the bank controls $\mathcal{C}$ (via $C$ ) up to $t_{0}+t_{1}(0)$. The above problem may then be written as

$$
\max _{R, \mathcal{C}} t^{K}(R, C)+p_{\lambda} R+\left(1-p_{\lambda}\right) \mathcal{C}-I
$$



Figure A.1: Inducing a pooling equilibrium.
subject to

$$
\left\{\begin{array}{l}
p_{a} R+\left(1-p_{a}\right) \mathcal{C} \leq a p_{a}+t_{A}^{F} \\
p_{b} R+\left(1-p_{b}\right) \mathcal{C} \leq b p_{b}+t_{B}^{F} \\
\mathcal{C} \leq t_{0}+t_{1}(0)
\end{array}\right.
$$

The geometry of the constraints is illustrated in Figure A.1. The intersection of the two IR constraints if both binding is (easily computed to be)

$$
\mathcal{C}^{*}=t_{0}+t_{1}(0)+\eta, R^{*}=a+t_{0}+t_{1}(1)-\frac{1-p_{a}}{p_{a}} \eta,
$$

with

$$
\eta \equiv \frac{p_{a} p_{b}(b-a)}{p_{a}-p_{b}}
$$

So $\mathcal{C}^{*}>t_{0}+t_{1}(0)$, and therefore the induced equilibrium must be with $\mathrm{IR}^{\mathrm{A}}$ binding. Indeed, suppose a policy induces an equilibrium like point ( $R^{\prime}$, $C^{\prime}$ ) in Figure A. 1 part (b); then Gov could induce a point like ( $R^{\prime \prime}, C^{\prime}$ ) at lower cost (lower $t^{K}$, all other transfers unchanged) while stil having all firms' rationality constraints satisfied. Hence, from $I^{\text {A }}$

$$
\mathcal{C}=C=t_{0}+t_{1}(0)-\frac{p_{a}}{1-p_{a}}\left(R-a-t_{0}-t_{1}(1)\right)
$$

Notice that the above equation, together with $\mathcal{C} \leq t_{0}+t_{1}(0)$, implies that $R \geq$ $a+t_{0}+t_{1}(1)$. Substituting and rearranging, bank's profits become

$$
\begin{aligned}
& t^{K}(\cdot)+a p_{\lambda}-I+\frac{1-p_{\lambda}}{1-p_{a}}\left(t_{0}+p_{a} t_{1}(1)+\left(1-p_{a}\right) t_{1}(0)\right)-\frac{R-a}{1-p_{a}}\left(p_{a}-p_{\lambda}\right) \\
&= t^{K}(\cdot)+a p_{\lambda}-I+p_{\lambda}\left(t_{0}+t_{1}(1)\right)+\left(1-p_{\lambda}\right)\left(t_{0}+t_{1}(0)\right) \\
&-\frac{p_{a}-p_{\lambda}}{1-p_{a}}\left(R-a-t_{0}-t_{1}(1)\right) \\
&= t^{K}(\cdot)+t_{\lambda}^{F}-\left(I-a p_{\lambda}\right)-\frac{p_{a}-p_{\lambda}}{1-p_{a}}\left(R-a-t_{0}-t_{1}(1)\right)
\end{aligned}
$$

And given that $R \geq a+t_{0}+t_{1}(1)$, an argument similar to the one justifying IR $^{\mathrm{A}}$ binding shows that the equilibrium induced by an optimal policy must have $R=a+t_{0}+t_{1}(1)$. On the other hand, bank's profits must be not les than prepolicy profits, which are $(1-\lambda)\left(b p_{b}-I\right)+=(1-\lambda) \zeta^{+}$. Hence the cost of policy, $t^{K}+t_{\lambda}^{F}$, is bounded below:

$$
\begin{equation*}
t^{K}+t_{\lambda}^{F} \geq I-a p_{\lambda}+(1-\lambda) \zeta^{+} \equiv t_{*} \tag{A1}
\end{equation*}
$$

This lower bound can be attained easily. For example, Gov may set $t_{\lambda}^{F}=0$ and $t^{K}(R, C)=t_{*}$ if $R \leq a$ and zero otherwise (which means saying to the bank, "If you charge $R \leq a$ I will cover your losses"). We thus have the following:

PROPOSITION 1: The cost of an optimal pooling policy is $t_{*}=I-a p_{\lambda}+$ $(1-\lambda) \zeta^{+}$.

## A.2. Inducing a Separating Equilibrium

Once more, we split analysis for the cases $\zeta \geq 0$ and $\zeta<0$.
Case $\zeta \geq 0$. We assume for now that $\left(R^{\mathrm{B}}, C^{\mathrm{B}}\right)=(b, 0)$. The idea is that since type $B$ dislikes collateral more than type $A$ (he has to pay it with a higher probability), the easiest way to keep him off A's contract is to let him away with no collateral (the $\mathrm{IC}^{\mathrm{B}}$ becomes harder to meet if $C^{\mathrm{B}}$ goes up). We verify later that it is actually more costly for Gov to reach any equilibrium with $\left(R^{\mathrm{B}}, C^{\mathrm{B}}\right) \neq(b, 0)$.

We then let $(R, C)=\left(R^{\mathrm{A}}, C^{\mathrm{A}}\right)$. Also, we use

$$
t^{\mathrm{B}}=t_{F, 0}(b, 0)+p_{b} t_{F, 1}((b, 0), 1)+\left(1-p_{b}\right) t_{F, 1}((b, 0), 0)
$$

and $t_{0}, t_{1}(s)$ and $\mathcal{C}$ will be as before. The transfer from Gov to the bank is now $t^{K}(\cdot)=t_{K, 0}((R, C),(b, 0))+p_{\lambda} t_{K, 1}(\cdot, 1)+\left(1-p_{\lambda}\right) t_{K, 1}(\cdot, 0)$.

So bank's profits are: on $B, t^{K}(\cdot)+b p_{b}-I$; and on A (if the firm signs contract ( $R, C$ ) )
$t^{K}(\cdot)+p_{a} R+\left(1-p_{a}\right)\left(C \wedge\left(t_{0}+t_{1}(0)\right)\right)-I=t^{K}(\cdot)+p_{a} R+\left(1-p_{a}\right) \mathcal{C}-I$.


Figure A.2: Inducing a separating equilibrium.

On the other hand, firm $X$ 's payoff from $(R, C)$ is

$$
p_{x}\left(x+t_{0}+t_{1}(1)-R\right)+\left(1-p_{x}\right)\left(t_{0}+t_{1}(0)-\mathcal{C}\right) .
$$

Therefore A's individual rationality and B's incentive compatibility constraints are respectively

$$
\begin{aligned}
& p_{a}\left(a+t_{0}+t_{1}(1)-R\right)+\left(1-p_{a}\right)\left(t_{0}+t_{1}(0)-\mathcal{C}\right) \geq 0 \\
& t^{\mathrm{B}} \geq p_{b}\left(b+t_{0}+t_{1}(1)-R\right)+\left(1-p_{b}\right)\left(t_{0}+t_{1}(0)-\mathcal{C}\right)
\end{aligned}
$$

which may be written as

$$
\begin{aligned}
& p_{a} R+\left(1-p_{a}\right) \mathcal{C} \leq t_{0}+p_{a}\left(a+t_{1}(1)\right)+\left(1-p_{a}\right) t_{1}(0) \\
& p_{b} R+\left(1-p_{b}\right) \mathcal{C} \geq t_{0}+p_{b}\left(b+t_{1}(1)\right)+\left(1-p_{b}\right) t_{1}(0)-t^{\mathrm{B}}
\end{aligned}
$$

Geometrically, the constraints are illustrated in Figure A.2. Bank's profits are

$$
t^{K}(\cdot)+(1-\lambda)\left(b p_{b}-I\right)+\lambda\left(p_{a} R+\left(1-p_{a}\right) \mathcal{C}-I\right),
$$

and as before the bank controls $\mathcal{C}$ (via $C$ ) up to $t_{0}+t_{1}(0)$. Viewing the bank's problem as one in $(R, \mathcal{C})$ with the additional constraint $\mathcal{C} \leq t_{0}+t_{1}(0)$, it is clear that it must be

$$
\begin{equation*}
t_{0}+t_{1}(0) \geq \mathcal{C}^{*} \tag{A2}
\end{equation*}
$$

with $\mathcal{C}^{*}$ as in Figure A.2, otherwise the bank's problem has empty feasible set. The intersection point drawn in the figure is

$$
\begin{aligned}
\mathcal{C}^{*} & =t_{0}+t_{1}(0)+\frac{p_{a}}{p_{a}-p_{b}}\left(p_{b}(b-a)-t^{\mathrm{B}}\right) \\
R^{*} & =a+t_{0}+t_{1}(1)-\frac{1-p_{a}}{p_{a}-p_{b}}\left(p_{b}(b-a)-t^{\mathrm{B}}\right)
\end{aligned}
$$

So (A2) is simply $t_{b}^{\mathrm{B}}(b-a)$; therefore Gov will set $t^{\mathrm{B}}=p_{b}(b-a)$, (A2) will hold with equality, and in equilibrium $(R, C)=\left(R^{*}, \mathcal{C}^{*}\right)$, i.e.,

$$
C=t_{0}+t_{1}(0), \quad R=a+t_{0}+t_{1}(1)
$$

Bank's profits are then

$$
t^{K}(\cdot)+(1-\lambda)\left(b p_{b}-I\right)+\lambda\left(t_{0}+p_{a}\left(a+t_{1}(1)\right)+\left(1-p_{a}\right) t_{1}(0)-I\right)
$$

and they are not less than prepolicy profits $(1-\lambda)\left(b p_{b}-I\right)$ iff

$$
t^{K}(\cdot)+\lambda\left(t_{0}+p_{a} t_{1}(1)+\left(1-p_{a}\right) t_{1}(0)\right) \geq-\lambda\left(a p_{a}-I\right)
$$

Since policy costs are $(1-\lambda) t^{\mathrm{B}}=(1-\lambda) p_{b}(b-a)$ plus the left hand side of the last expression, they are bounded below by

$$
(1-\lambda) p_{b}(b-a)-\lambda\left(a p_{a}-I\right)=t_{*},
$$

exactly the same as for pooling (here $\zeta \geq 0$ so $\zeta=\zeta^{+}$).
In this case this lower bound can be attained, for example by setting $t^{K}=$ $t_{0}=t_{1}(0)=0, t^{\mathrm{B}}=p_{b}(b-a)$, and

$$
t_{1}(1)=-\frac{a p_{a}-I}{p_{a}} .
$$

Here Gov says to firms: "If you sign contract $(b, 0)$ you get $t^{\mathrm{B}}$; if you sign any other contract you pay $-t_{1}(1)$ in case of success." We may thus state:

PROPOSITION 2: When $\zeta \geq 0$, the cost of an optimal separating policy is the same as the cost of an optimal pooling policy, $t_{*}$.

Before we turn to the case $\zeta<0$ we should confirm that the assumption that $\left(R^{\mathrm{B}}, C^{\mathrm{B}}\right)=(b, 0)$ was without loss of generality. Indeed, for the bank to improve upon $(b, 0)$ it must be (with self-evident undefined notation)

$$
p_{b} R^{\mathrm{B}}+\left(1-p_{b}\right)\left(C^{\mathrm{B}} \wedge\left(t_{0}^{\mathrm{B}}+t_{1}^{\mathrm{B}}(0)\right)\right) \geq b p_{b},
$$

that is

$$
\begin{equation*}
C^{\mathrm{B}} \wedge\left(t_{0}^{\mathrm{B}}+t_{1}^{\mathrm{B}}(0)\right) \geq \frac{p_{b}\left(b-R^{\mathrm{B}}\right)}{1-p_{b}} . \tag{A3}
\end{equation*}
$$

The incentive compatibility constraint for B becomes

$$
\begin{aligned}
& p_{b}\left(b+t_{0}^{\mathrm{B}}+t_{1}^{\mathrm{B}}(1)-R^{\mathrm{B}}\right)+\left(1-p_{b}\right)\left(t_{0}^{\mathrm{B}}+t_{1}^{\mathrm{B}}(0)-\left[C^{\mathrm{B}} \wedge\left(t_{0}^{\mathrm{B}}+t_{1}^{\mathrm{B}}(0)\right)\right]\right) \\
& \quad \geq p_{b}\left(b+t_{0}^{\mathrm{A}}+t_{1}^{\mathrm{A}}(1)-R^{\mathrm{A}}\right)+\left(1-p_{b}\right)\left(t_{0}^{\mathrm{A}}+t_{1}^{\mathrm{A}}(0)-\mathcal{C}\right)
\end{aligned}
$$

but from (A3) one derives

$$
\begin{aligned}
t_{0}^{\mathrm{B}}+p_{b} t_{1}^{\mathrm{B}}(1) & +\left(1-p_{b}\right) t_{1}^{\mathrm{B}}(0) \\
& \geq p_{b}\left(b+t_{0}^{\mathrm{A}}+t_{1}^{\mathrm{A}}(1)-R^{\mathrm{A}}\right)+\left(1-p_{b}\right)\left(t_{0}^{\mathrm{A}}+t_{1}^{\mathrm{A}}(0)-\mathcal{C}\right) \\
& =t_{F, 0}(b, 0) \quad \text { in the equilibrium we found before. }
\end{aligned}
$$

The other constraints being unaffected, we conclude that implementing a separating equilibrium via a ( $\left.R^{\mathrm{B}}, C^{\mathrm{B}}\right) \neq(b, 0)$ is always (weakly) more costly for Gov than with $(b, 0)$.

Case $\zeta<0$. In this case an efficient separating equilibrium has only Atypes financed; B should sign no contract. Notation is as before, except that now

$$
t^{\mathrm{B}}=t_{F, 0}(\emptyset),
$$

and without loss of generality we are taking $t_{F, 1}(\emptyset, s)=0$.
Bank's profits on A are $t^{K}(\cdot)+p_{a} R+\left(1-p_{a}\right) \mathcal{C}-I$; and given $t^{\mathrm{B}} \geq 0$, the individual rationality and incentive compatibility constraints are now

$$
\begin{aligned}
& p_{a}\left(a+t_{0}+t_{1}(1)-R\right)+\left(1-p_{a}\right)\left(t_{0}+t_{1}(0)-\mathcal{C}\right) \geq t^{\mathrm{B}} \\
& t^{\mathrm{B}} \geq p_{b}\left(b+t_{0}+t_{1}(1)-R\right)+\left(1-p_{b}\right)\left(t_{0}+t_{1}(0)-\mathcal{C}\right)
\end{aligned}
$$

that is,

$$
\begin{aligned}
& p_{a} R+\left(1-p_{a}\right) \mathcal{C} \leq t_{0}+p_{a}\left(a+t_{1}(1)\right)+\left(1-p_{a}\right) t_{1}(0)-t^{\mathrm{B}} \\
& p_{b} R+\left(1-p_{b}\right) \mathcal{C} \geq t_{0}+p_{b}\left(b+t_{1}(1)\right)+\left(1-p_{b}\right) t_{1}(0)-t^{\mathrm{B}}
\end{aligned}
$$

The geometry of the problem is as in the previous case, and the last picture still applies as is. As before the bank chooses $\mathcal{C}$ subject to $\mathcal{C} \leq t_{0}+t_{1}(0)$, and again (A2) must hold. In the present case the intersection $\left(\mathcal{C}^{*}, R^{*}\right)$ is (with $\eta$ as on p. 18)

$$
\mathcal{C}^{*}=t_{0}+t_{1}(0)+\eta-t^{\mathrm{B}}, \quad R^{*}=a+t_{0}+t_{1}(1)-\frac{\eta}{p_{a}}
$$

So (A2) is $t^{\mathrm{B}} \geq \eta$; Gov will then set

$$
t^{\mathrm{B}}=\eta
$$

(larger than before), and $(R, C)=\left(R^{*}, \mathcal{C}^{*}\right)$ in equilibrium. Bank's profits are then

$$
\begin{aligned}
& t^{K}(\cdot)+\lambda\left(p_{a} R+\left(1-p_{a}\right)\left(t_{0}+t_{1}(0)\right)-I\right) \\
& \quad=t^{K}(\cdot)+\lambda\left(t_{0}+p_{a} t_{1}(1)+\left(1-p_{a}\right) t_{1}(0)\right)+\lambda\left(a p_{a}-I-\eta\right)
\end{aligned}
$$

Also, the cost of policy is $t^{K}(\cdot)+\lambda\left(t_{0}+p_{a} t_{1}(1)+\left(1-p_{a}\right) t_{1}(0)\right)+(1-\lambda) t^{\mathrm{B}}$. And as before bank's profits must be larger than prepolicy profits, which are zero in this case; hence policy costs are bounded below by

$$
(1-\lambda) t^{\mathrm{B}}+\lambda\left(\eta-\left(a p_{a}-I\right)\right)=\eta-\lambda\left(a p_{a}-I\right)
$$

Again the bound can be attained, for example by setting $t^{\mathrm{B}}=\eta$ and all other transfers equal to zero except $t_{1}(1)$ given by

$$
t_{1}(1)=\frac{1}{p_{a}}\left(\eta-\left(a p_{a}-I\right)\right) .
$$

Note, by the way, that this implies that equilibrium collateral is zero in this case. We have thus reached to following:

PROPOSITION 3: When $\zeta<0$, the cost of an optimal separating policy is $\eta-\lambda\left(a p_{a}-I\right)$.

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[^1]:    ${ }^{1}$ Timing is as follows: investment is needed at date 0 , returns accrue at date 1 , and agents have no time preference.

[^2]:    ${ }^{2}$ For example, in the case of Southern Italy: (1) The loans/deposit ratio is around half as much as in the North; (2) long-term debt is almost half of total, while it is one third in the North; and (3) the spread between borrowing and lending interest rates is about $15 \%$ higher than in the North for long term debt; for short-term loans it is more than $50 \%$ higher. Cf. Banca d'Italia, Bollettino Statistico.

[^3]:    ${ }^{3}$ Payoffs from a debt contact R should be written $u_{X}(R)=p_{x} \max \{x-R, 0\}$ and $v(R$; $X)=p_{x} \min \{R, x\}-I$, respectively. Then the firm never refuses any contract. This would deprive the model of its interest, which is to capture the fact that less risky firms drop out of the market if $R$ is too high. To motivate the payoff we use in the text one could imagine a (linear) bankruptcy penalty. Alternatively, one might assume that in case of success, if $R>$ $x$ firm $X$ can and will borrow $R-x$ to fully meet its current obligations. This second assumption seems reasonable: borrowing will be possible because banks trust successful firms. On the other hand, to satisfactorily incorporate this intuition into a formal model one should have an infinite horizon. At this stage we prefer to interpret our payoffs as a brute force device to model the fact that firms of type $X$ will not accept a contract with $R>x$.

[^4]:    ${ }^{4}$ Separation would result also for lower levels of wealth, but to be more specific on this would only complicate notation.

