OPEN SOURCE WITHOUT FREE-RIDING

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1. Introduction

Several issues relating to open source software are discussed by academics, businessmen and politicians. We focus here on the somewhat surprising fact that "otherwise fierce competitors are demonstrating that they can benefit from embracing the Open Source philosophy of sharing work" (*Business Week*, January 2005). [1] Incidentally, perhaps even more surprisingly, the potential of Open Source is now so much taken for granted in the business community that *not* embracing it may be seen as a major issue. [2]

For software as for any other good, R&D output is non-rival. The exciting thing about Open Source (OS) is that, as hackers like to put it, "Each contributes a brick and each gets back a complete house in return" (cfr. Prasad (2001)); problem is that the OS house is yours even if you do not spare your brick. So why bother at all? And of course if no-one puts her brick there is no house to share. The purpose of this paper is to derive conditions under which this free-riding problem is overcome.

The idea we start with is close to Linus Torvalds' explanation of the success of the Open Source innovation process (Torvalds invented Linux in 1990): "Much software will be developed this way. It's especially good for infrastructure –stuff that affects everybody" (*Business Week* cited above). We build on the intuition that individual investment in OS development of a common input whose improvements raise the productivity of the other firm-specific inputs may be profitable even if it

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Date: April 19, 2012.
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[1] The quotation refers to the firms contributing to the Linux computer operating system, see `linux-foundation.org`.
[2] Cfr. "Apple's Two Biggest Risks", by Eric Jackson at `thestreet.com`, 2 June 2010.

does not directly give an edge to the single contributor (for any one's effort increases competitors' potential as well as its own), because it determines an advantage to the industry as a whole, hence to its average firm.

The argument is in principle not confined to software, and concerns common inputs more generally. We analyze and compare two alternative organizations for the production of such an input: one decentralized, regulated by a stylized Open Source License; the other centralized, where production is carried out by a monopolistic firm under a proprietary license.

The noticeable feature in actual OS projects is that there are often promoting firms which might sign closed R&D sharing agreements, and instead leave the research platforms totally open. Our conjecture is that this must be due to the structure of research production: progress is faster if all potential contributions are exploited and cumulated, and this requires complete spreading of the knowledge base. The situation is similar for industrial and scientific research: each single developer/researcher may have a first path-breaking idea on a product or research line, but this is more often then not followed by other ones which are comparatively less important. Thus a fixed research team is bound to quickly exhaust strongly innovative ideas on a given project. On the other hand if the technical information which one has to inspect in order to think about and innovations is universally accessible then new contributors can always come up with their "first ideas", making the chain of substantial improvements much longer. The formal translation of this argument is that in firm-based production of new knowledge there are strongly decreasing returns to scale, and this will be our central assumption on the research cost structure.

What we show (see Section 3) is that, in the presence of decreasing returns to knowledge production, in a monopolistically competitive industry where many firms compete but preferences for their differentiated products are strong, the productivity-enhancing effect of the common input generates a situation where research under a General Public License –the license that underlies the OS production mode– is not only non-zero, but may be more intense than under a proprietary monopolistic license. The role of market power in the argument is that the firm contributing to open source research does increase her competitors' potential, but this effect is counter-balanced by the fact that her customers have intense preference for her own product. The intuition for the result is given is section 3.

Formally we study two-stage games where firms pursue cost-reducing research in the first stage and engage in Bertrand competition in a spatial model à la Hotelling in the second. So the paper is linked to the literature on R&D spillovers, cartels and joint ventures, where the general structure of the models studied is the same as ours, with research conducted in the first stage and product competition in the second. In this line of research the notable contribution by Amir et al. (2003) reviews and extends earlier research. The essential difference in our setting lies in the structure of R&D expenses: in the cited literature a fraction of *all* of a firm's R&D results spill over to the others; our firms on the other hand may conduct *private* research on firm-specific inputs, whose results remain private, and at the same time, as an independent choice, they may voluntarily contribute to research on shared inputs whose results are common property.

The result of this paper complements two sets of results. One is in the work of Bessen and Maskin (2006), who find, in a dynamic model emphasizing the sequential nature of innovation focused on the trade-off between research and imitation, that under some conditions a no-patent regime may be more favorable to innovation than a patent-protected system. The other is contained in the extensive research by Boldrin and Levine who analyze several aspects of knowledge production, the main assumption common to their work and the present paper being that of diminishing returns. The model of theirs which is most easily compared to this paper is perhaps Boldrin and Levine (2009), where a monopolist is introduced in an otherwise competitive market; the result is that the addition reduces welfare and the quality of research.

In the sequel, the next section presents the model, section 3 the result of the paper and section 4 contains concluding comments.

2. The Model

We study subgame perfect equilibria of two-stage games, building on the "circular city" version of the Hotelling model due to Salop (1979), extending it to the case of firms producing at different unit costs.

Going backwards, in the second stage there is a price-setting game among n firms $i = 1, \dots, n$ in a spatial model. Firms are uniformly spread along a unit-length circle, and produce a good at unit cost c_i whose value, high or low, is stochastically determined by research investments made in the first stage. The good is sold to a unit mass of consumers who buy a unit each from the firm which they find more convenient on the basis of the firms' selling prices p_i and of their unit transportation cost $t > 0$. At the time of its price setting decision, firm i knows its own cost and the expected proportion of low-cost firms (deducible from known first-stage actions), but we assume that it does not observe her neighbors' type, i.e. their realized cost.

In the first stage firms invest to pursue a cost-reducing innovation which would increase profits in stage two. The firms' production technology is not formally specified, but is thought of as involving two types of inputs: firm-specific and shared ones. We then model research as consisting of two types of efforts, directed at the two types of inputs and resulting in two numerical indexes of research output, x_i and z_i respectively; the first-stage actions are $(x_i, z_i),\ i = 1, \dots, n$, which influence the probability of cost reduction. Firm-specific x_i is assumed for simplicity to be produced at constant, unit marginal cost; research on the common input has convex production costs $c(z_i)$.

We are interested in the consequences of imposing a General Public License (GPL) on the common input —of course a stylized version of it.[3] The GPL imposes first not to impose the restrictions of use, modification and distribution of the usual proprietary licenses; and second, to release the modified product under GPL in turn. So the whole sequence of developments of a product released under GPL will be *Open Source.* Thanks to openness of the knowledge base the GPL induces a process of research on innovation where contributions typically come from many independent research units, usually self-coordinating in agreed-upon 'meeting places'. To get output z_i the firm contributes $c(z_i)$ to the community of developers (some often belonging to the firm itself). Therefore GPL makes the output of research on the common input, which is non-rival by nature, a public good by rendering it

[3]Actual licenses contain subtle law statements which are beyond our understanding. The GPL, together with more than a hundred other existing 'compatible' licenses, is at `gnu.org/licenses/license-list.html`. The type we describe is close in spirit to the GPL version 2.

non-excludable.[4] Individual efforts add up and all exploit the cumulative result. The problem, which motivates this paper, is that by not investing at all a firm can still appropriate the result of the others' efforts.

The proprietary system we contrast to open-source mode is one where the common input is provided by an external patent-protected monopolist, who sells it to the firms for profit and is the only one who can conduct research on its product. Comparison of the latter with Open Source production mode gives our interpretation of the Linux-versus-Window development.

We next describe these alternative institutional settings: first the one with n competing firms with GPL'd common input, then the other with the same firms but where research on common input is provided by an external patented monopolist (player 0). The second stage is common to both environments, and with it we start.

2.1. *Second Stage*. The n firms are evenly located around the circle, and in second-stage price competition a fraction q_l have low cost and the rest, $q_h = 1 - q_l$, have high cost; we let $q = (q_l, q_h)$. These relative proportions are endogenously determined by research investments in the first-stage of the game, where players take into account the equilibrium continuation they anticipate. In the second stage q is given and known to the firms.

Firm i produces at cost $c_i \in \{c_l, c_h\}$, where $c_h = c,\ c_l = (1-\delta)c$, δc being the cost reduction enjoyed by the nq_l firms who have succeeded in innovating. We let $\bar{c} = q_l c_l + q_h c_h$ and $\bar{p} = q_l p_l + q_h p_h$, with p_l, p_h denoting prices charged by low- and high-cost firms. Given private types, $\bar{p}$ is also the common expected price of i's competitors, so that firm i's demand is derived as in Tirole (1988), ch.7:

$$D_i(p_i) = \frac{\bar{p} - p_i + t/n}{t}\,. \tag{1}$$

[4] GPL actually admits non-disclosure, on the part of a firm, of the results of its own research on the GPL'd input. We are assuming that the amount of research which can be pursued in isolation from the community of developers by a single firm on an open source project is negligible. As a matter of fact, relevant non-disclosures would be deducible by the competitive edge they would create, hence quite loudly criticized by the community. We know of no such episode.

Demand is really $\max\{D_i, 0\}$, but Assumption 1 below will guarantee that the lower bound is inactive. Firm i solves

$$\max_{p_i} \pi_i = \max_{p_i} \, (p_i - c_i) \, D_i(p_i) \, . \tag{2}$$

Equilibrium profits π_l and π_h of the two types of firms will depend on the cost vector $C = (c_l, c_h)$ and on q, but as the next Lemma shows dependence on C is only through δc. To ensure that both types of firms stay in the market with positive demand we will need $t/n \geq q_l \delta c/2$; we make the following stronger assumption which makes also first-stage investments in research profitable, and implies the one needed here for n large:

Assumption 1. *t/n^2 is bounded away from 0 as n grows.*

This is the assumption we have about fidelity, that consumers have strong preferences for the firms' differentiated products.

Lemma 1. *Under Assumption 1, for large n both types of firms stay in the market, and second-stage equilibrium profits depend on costs only through δc and are given by*

$$\pi_l(\delta c, q) = t\left(\frac{1}{n} + q_h \frac{\delta c}{2t}\right)^2 \quad \textit{and} \quad \pi_h(\delta c, q) = t\left(\frac{1}{n} - q_l \frac{\delta c}{2t}\right)^2 . \tag{3}$$

Proof. Equating derivative of π_i in (2) to zero one obtains $p_i = \big(\bar{p} + c_i + t/n\big)/2$; by taking expectations one gets $\bar{p} = \bar{c} + t/n$; and substituting this in the expression for p_i gives

$$p_i = \frac{c_i + \bar{c}}{2} + \frac{t}{n} \, .$$

Since $c_h - c_l = \delta c$ we have $(c_h + \bar{c})/2 = c_h - q_l \delta c/2$ and $(c_l + \bar{c})/2 = c_l + q_h \delta c/2$. So, assuming symmetry within groups, one has

$$p_l = c_l + \frac{t}{n} + q_h \frac{\delta c}{2} \, , \quad p_h = c_h + \frac{t}{n} - q_l \frac{\delta c}{2} \, . \tag{4}$$

In order for high cost firms to stay in the market we need $t/n \geq q_l \delta c/2$ for any q_l, or $t/n \geq \delta c/2$, which is guaranteed by Assumption 1 for large n.

Now from (4) $p_h - p_l = \delta c/2$; on the other hand $p_h - \bar{p} = q_l(p_h - p_l)$ and $\bar{p} - p_l = q_h(p_h - p_l)$; therefore from (1) equilibrium demands are

$$D_l = \frac{1}{n} + \frac{q_h \delta c}{2t} \quad \textit{and} \quad D_h = \frac{1}{n} - \frac{q_l \delta c}{2t} \,,$$

whence the given expression for equilibrium profits follows. □

2.2. *First Stage, Common Input Developed under GPL.* Here firms invest to increase the probability of an innovation which starting from a symmetric situation $c_i = c$ for all i gives the innovator a cost advantage of δc, bringing down his cost from c to $(1-\delta)c < c$. Research output for firm-specific and common input being x_i and z_i, moves for i in this stage are pairs $s_i = (x_i, z_i)$. A profile of moves will be denoted by $s = (s_1, \ldots, s_n)$ as usual.

Our goal is to find the equilibrium total research output $Z \equiv \sum_i z_i$. This we will compare, in section 3, with the quantity produced by the monopolist in the alternative patent-protected setting.

To model the fact that the common input is subject to a GPL we take the probability q_i that i will be low-cost as being influenced by individual x_i and *cumulative* Z, the latter reflecting the fact that the good produced under GPL becomes public. We also want to capture the 'stuff-that-affects-everybody' aspect, that is, complementarity of the quality of the two types of inputs (for example, better computers increase the probability that research on wine production result in a better product). A specification of q_i that incorporates these aspects is the following, where $X = \sum_i x_i$ and f is an increasing function bounded above by 1 (Assumption 2 below): [5]

$$q_i(s) = \frac{x_i}{X/n} f(Z) \,. \tag{5}$$

In the literature on rent–seeking games (see e.g. Baye et al. (2003)) the probability of winning is based on the fraction of invested resources; here there is not a single winner, and we take the probability of the realization of a cost reduction dependent

[5] Strictly speaking q_i should not be called a probability because it can be larger than 1. The situation we have in mind is one with many similar firms whose investments are all in the same range, so that this measure can be bounded; we omit normalization for simplicity.

on individual investment relative to the average; [6] on top of it we add $f(Z)$ which gives a role to investment in the common input. Note that an increase in z_i also raises q_j. Our assumption on f is the following:

Assumption 2. *f is concave, increasing from $f(0) \in [0, 1/2)$ to 1 as Z goes from zero to infinity.*

The value of $f(0)$ will ensure that investment in the common input is positive; the fact that $f < 1$ will guarantee that the fraction of low-cost firms is actually in $(0, 1)$. Indeed, given q_i, $i = 1, \ldots, n$, the fraction of low cost firms will be their average:

$$q_l = q_l(Z) = n^{-1} \sum\nolimits_i q_i = f(Z)\,,$$

and expected number of low-cost firms will be $\sum_i q_i = nq_l$. Notice that $q_i > q_l$ iff $x_i/X > 1/n$.

First-stage expected payoffs are then, with q_i and second-stage profits given by equations (3) and (5),

$$u_i(s) = q_i(s)\pi_l + (1 - q_i(s))\pi_h - x_i - c(z_i)\,.$$

As mentioned above firm-specific x_i is always assumed to be produced at constant, unit marginal cost (this is not substantial). On the other hand, on research costs of the shared input we make the following

Assumption 3. *The function c is increasing convex along with its derivative c', and $c'(0) = 0$.*

We can now state

Lemma 2. *Under Assumptions 1 to 3 the research output produced under GPL in symmetric equilibrium, which will be denoted by Z^{GPL}, is the positive number defined by the relation*

$$A = \frac{c'(n^{-1}Z)}{f'(Z)\,(1 - 2f(Z))}\,, \quad \text{where } A = (\delta c)^2/4t\,. \tag{6}$$

[6] This is intended to capture the fact that for a firm to be at an advantage with respect to the others, it must invest more than them. It is true that investment generally entails absolute cost reductions, but in our case all that matters is relative advantage. The results of the paper also hold if c is assumed to decrease with X.

Z^{GPL} *increases with* δ *and with the number of firms in the market,* n.

Proof. Letting $\psi_i = nx_i/X$ and noting that $q_i = \psi_i q_l$ one derives that

$$(7) \qquad u_i(s) = \frac{t}{n^2} + \frac{\delta c}{n}\, q_l(\psi_i - 1) + \frac{(\delta c)^2}{4t}\left[\psi_i q_l - q_l^2(2\psi_i - 1)\right] - x_i - c(z_i)\,.$$

Setting partial derivative of u_i with respect to z_i equal to zero gives, recalling that $\partial q_l/\partial z_i = f'(Z)$,

$$\frac{2c'(z_i)}{\delta c f'} = \frac{2}{n}(\psi_i - 1) + \frac{\delta c}{2t}\big(\psi_i - 2(2\psi_i - 1)q_l\big)\,;$$

and since $\sum_i \psi_i = n$, by summing over i one obtains, after substituting ψ_i and rearranging, the (subgame perfect) equilibrium condition

$$A \;=\; \frac{n^{-1}\sum_i c'(z_i)}{f'(Z)\,(1 - 2f(Z))}\,.$$

Notice from this that a symmetric equilibrium exists. Concentrating on this equilibrium, we see that although the FOC is really $\partial u_i/\partial z_i \le 0$, the assumption $f(0) < 1/2$ ensures that $\partial u_i/\partial z_i > 0$ at $Z = 0$, so the equation displayed above holds, and equilibrium $Z > 0$. In fact by symmetry $z_i = n^{-1}Z$ for all i, so the equation is exactly (6). That Z^{GPL} increases with δ and n is obvious from (6). □

Remark: Free Riding. The fact that $Z^{GPL} > 0$ says that there is no free riding in this equilibrium: for each i, $z_i > 0$. The possibility of free riding emerges if we take the following more general specification for q_i than in (5):

$$q_i(s) = \;(1-\eta)\frac{x_i}{X/n} \;+ \eta\,\frac{x_i}{X/n}\,f(Z)\,, \quad \eta \in [0,1]\,.$$

It is easy to see that for low η the symmetric equilibrium has $Z = 0$. Since

$$\frac{\partial}{\partial \eta}\frac{\partial q_l}{\partial Z} = f'(Z) > 0\,,$$

η measures the strength of the effect of Z on productivity of research. No free riding results if this effect is strong enough. By taking the above more general specification for q_i, which we avoid in the text for ease of exposition, all the results we present hold with the qualification *"for η close enough to 1"*.

2.3. *First Stage, Common Input Developed by Outside Monopolist.* Here we study the $(n+1)$–player model where besides the n firms around the circle there is a monopolist, player 0, who sells research output z to the n competitors. Our goal is again to find equilibrium output. This is still non-rival but non-excludability is eliminated by law in this proprietary system, the public nature of the common input being suppressed by the imposition of a proprietary license.

For the n competitors $i = 1, \ldots, n$, stage two (price competition) is still as in the previous case, with profits depending on δc and q given by Lemma 1. But there are two changes in the first stage. The first is in $q_i(s)$, the probability with which i will be low-cost in stage two (cfr. (5)): it is now no longer cumulative Z which enters the formula, but just the amount z_i which firm i acquires from the monopolist; in other words we now have

$$q_i(s) = \frac{x_i}{X/n} f(z_i) , \tag{8}$$

and as a consequence $q_l = n^{-1} \sum_i q_i = \sum_i \frac{x_i}{X} f(z_i)$.

The second difference is that firm i no longer produces z_i, but buys it at the price p the monopolist sets. Thus firm i's first-stage payoff is (compare (7))

$$u_i(s) = q_i \pi_l + (1 - q_i)\pi_h - x_i - p z_i , \tag{9}$$

where now q_i is to be read from equation (8) above.

Consider now the monopolist. Given the non-rivalry of his product, whatever he produces for one firm can be re-used for all n. Since in principle the various firms may demand different amounts of the monopolist's service, he will have to produce the highest required; but with that he is able to serve the whole market. In other words, he produces $\max_i z_i$ and sells $\sum_i z_i$. In the case of software for example, we are talking of the 'base' and the 'professional' version with more features, each user buying the version ('quantity of code') which better suits her needs. To ensure a common scale in the comparison with the GPL model we assume that the monopolist firm has the same cost function $c(\cdot)$ used above. In the symmetric equilibrium we shall consider $x_i = n^{-1}X$ and $z_i = z$ for all i. In such an equilibrium the monopolist produces z and sells nz, so that his problem becomes

$$\max_z \; nz\,p(z) - c(z)\,. \tag{10}$$

where p is the demand function. Let now z^{mon} the z the research output produced by the monopolist in symmetric equilibrium. In the next section the result of the paper is presented, where this quantity is compared to the Z^{GPL} defined by equation (6) on page 8. We have

Lemma 3. *The research output produced by the monopolist in symmetric equilibrium, z^{mon}, is defined by the following condition (where $A = (\delta c)^2/4t$ as on p. 8):*

$$A\left[1 + z\frac{f''(z) - 2(1+\gamma-2f)^{-1}f'^2}{f'(z)}\right] = \frac{n^{-1}c'(z)}{f'(z)\big(1+\gamma-2f(z)\big)}\,. \tag{11}$$

Proof. Start with the firm's problem of maximizing (9). Using again ψ_i from page 8, so that $q_i = \psi_i\, f(z_i)$, substitution from second-stage profits (3) leads to the following expression for $u_i(s)$:

$$u_i(s) = \frac{t}{n^2} + \frac{\delta c}{n}\left[\psi_i f(z_i) - q_l\right] + \frac{(\delta c)^2}{4t}\left[\psi_i f(z_i)(1-2q_l) + q_l^2\right] - x_i - pz_i\,.$$

The FOC with respect to z_i gives, fixing the other firms' investments z_{-i}, price p as function of z_i, whose inverse is firm i's demand of z_i at price p. We then set $\partial u_i/\partial z_i = 0$; noting that $\partial q_l/\partial z_i = n^{-1}\psi_i f'(z_i)$, this yields

$$p = \psi_i f'(z_i)\left[\frac{(\delta c)^2}{4t}\Big(1 - 2q_l(z_i, z_{-i})\,\frac{n-1}{n} - 2\frac{\psi_i f(z_i)}{n}\Big) + \delta c\,\frac{n-1}{n^2}\right]. \tag{12}$$

We check here that the above FOC is sufficient for a maximum of u_i; one has in fact

$$\frac{1}{\psi_i}\frac{\partial^2 u_i}{\partial z_i^2} = f''\,[\cdot] - f'\frac{(\delta c)^2}{2nt}\Big((n-1)q_l' + \psi_i f'\Big)\,, \tag{13}$$

where the bracketed expression is the one of (12), positive whenever (12) holds; our assertion then follows by recalling that f' and q_l' are positive while f'' is negative.

Given that first order conditions are sufficient the symmetric equilibrium is found by substituting them into the monopolist problem (10), which is characterized by the FOC

$$p + zp' \leq n^{-1}\, c'(z)\,. \tag{14}$$

Given $f(0) < 1/2$ (from assumption 2) it is $p(0) > 0$, whence the monopolist will produce positive z and meet the FOC with equality. By symmetry $\psi_i = 1$ all i and $q_l = f(z)$, so that p and p' read

$$\begin{aligned} p(z) &= \, Af'(z)\,[1+\gamma-2f(z)]\,, \\ p'(z) &= \, A\left[f''(z)\big(1+\gamma-2f(z)\big)\; - 2f'(z)^2\right], \text{ with } \gamma = \frac{4t}{\delta c}\frac{n-1}{n^2}\,. \end{aligned}$$

Using the above expressions for p and p' the FOC (14) (with equality) results to be as asserted in (11). □

3. The Equilibrium Research Output

The proposition stated below, on the comparison of the research outputs defined in Lemmas 2 and 3 for the alternative institutional settings, is the result of the paper. It says that under the maintained assumptions, in symmetric equilibrium for n large the GPL economy produces more research on the common input than that provided by a patent-protected monopolist.

Proposition. *Under Assumptions 1-3, for n large enough, in symmetric equilibrium, the research output Z^{GPL} produced under GPL is larger than the z^{mon} produced by the monopolist.*

Proof. The two quantities are defined in Lemmas 2 and 3. First observe that assumptions 2 and 3 imply that the right members of equation (6) and (11) are increasing in Z and z respectively.

Since $f'' < 0$, the left member of (11) is smaller than A; therefore z^{mon} is smaller than the z determined by

$$A = \frac{n^{-1}c'(z)}{f'(z)\big(1+\gamma-2f(z)\big)}\,. \tag{15}$$

But

$$\frac{n^{-1}c'}{f'\big(1-2f\big)} - \frac{n^{-1}c'}{f'\big(1+\gamma-2f\big)} = \frac{1}{n}\,\frac{c'}{f'\big(1-2f\big)}\,\frac{1}{1+\frac{1-2f}{\gamma}}$$

is small for n large (for $\gamma \to \infty$ as $n \to \infty$), so z^{mon} is also smaller than the z^o determined by

$$A = \frac{n^{-1}c'(z^o)}{f'(z^o)\big(1-2f(z^o)\big)} \equiv \mathcal{M}(z^o)\,.$$

On the other hand, equilibrium Z^{GPL} is defined by equation (6). Denoting its right member by $\mathcal{C}(Z)$ we have $A = \mathcal{C}(Z^{GPL})$. But assumption 3 implies (elementary calculus) $n^{-1}c'(z) \geq c'(n^{-1}z)$. Thus $A = \mathcal{C}(Z^{GPL}) \leq \mathcal{M}(Z^{GPL})$; and since $\mathcal{M}(z^o) = A$ and $\mathcal{M}$ is increasing, $z^o \leq Z^{GPL}$. Result is thus proved because we already know that $z^{mon} < z^o$. □

The economically substantial hypotheses driving this result are Assumption 3, which says that marginal cost should grow at increasing speed (like in $c(z) = z^\alpha$ with $\alpha \geq 2$); and Assumption 1, which together with n large depicts a market where firms are numerous but their market power is non-negligible.

The cost-function structure enters in the numerators of the right members of (6) and (11); specifically, Assumption 3 amounts to requiring that $nc'(z/n) < c'(z)$ for each n. To motivate it observe that, in symmetric equilibrium, under GPL there is a common amount $z^{GPL} = Z^{GPL}/n$ produced by each firm, the social cost is $nc(z^{GPL})$ and the amount available for use by each firm is nz^{GPL}; on the other hand, under monopoly there is a common amount z^{mon} purchased by each firm, the social cost is $c(z^{mon})$ and the amount available for use by each firm is z^{mon}. The assumption then says that at the margin it is cost-efficient to decentralize non-rival production.

The intuition for the Proposition 3 is the following. First, research investment is of the same order of magnitude in shared and proprietary production environments. The reason is that given intrinsic non-rivalry of research output, under both scenarios each enjoys n times what he produces (where n are the firms contributing to OS or the monopolist's customers): in the first case each of the firms using the shared input, owing to the public-good nature of research output under GPL; in the second case the monopolist producing the non-rival patented research on the input, because she sells the same thing n times. Within this order of magnitude, two opposite forces influence the difference: one is the inappropriability of effort under GPL (a business-stealing effect), which in our case potentially generates free-riding but is mitigated by customers' fidelity; the other is a monopoly output restriction effect, due to the fact that in OS mode firms get research output at cost price, while with the monopolist around they pass a mark-up to him. The balance leans in favor of shared production mode if knowledge accumulation has strongly decreasing returns to scale.

One may see the two opposite forces at work by comparing the two relevant FOC's, equation (6) for OS and (11) for proprietary models: the monopoly output restriction effect is reflected in the extra negative term appearing in the left member of (11); the inappropriability effect shows up in the γ term in the right member of the same equation.

4. Conclusions

We have discussed R&D on shared inputs which affect productivity of firm-specific ones, and shown that the amount of innovation on this kind of inputs is higher under a General Public License (GPL, the Open Source license) than under a proprietary license held by a monopolist if marginal research costs grow rapidly and the industry comprises many firms with non-negligible market power (due to customers' strong preferences for their differentiated goods). The phenomenon described in the result has been actually observed in the last decade in the software industry, first for the Linux computer operating system and more recently in mobile devices. The industry structure in the latter case, with strong competition and market power, satisfies quite remarkably our central assumption. Our result then seems to be saying that Open Source is thriving where it works best. As argued in the introduction the conditions under which the possibility arises are in principle not confined to software, and it would be interesting to identify cases where it may be a promising route for research.

However, Open Source development can unfold only if the primary innovation is released under a GPL. This is something the inventor may choose to do, as it has been the case for Linux or Apache and more recently for Android; but more often the inventor imposes a proprietary license, for immediate benefit or miscalculation. Thus in the cases where the conditions for efficiency of Open Source development hold, a patent policy trade-off emerges between fostering initial inventions and speeding up product development: imposing GPL by law would abate primary inventions, but proprietary licenses would slow down subsequent innovation. A policy route which may be worth exploring in such contexts is the one hinted at by Kremer (1998), which consists of granting patents to inventors, and then proceeding with patent buyouts on the part of Government when this is judged beneficial, followed by release the patent's content under a GPL if the conditions suggested in this paper hold.

References

Amir, Rabah, Igor Evstigneev and John Wooders (2003): Noncooperative versus Cooperative R&D with Endogenous Spillover Rates, *Games and Economic Behavior* **42**: 183–207

Baye, Michael R. and Heidrun C. Hoppe (2003): The strategic Equivalence of Rent-Seeking, Innovation, and Patent-Race Games, *Games and Economic Behavior* **44**: 217–226

Bessen, James and Eric Maskin (2006): "Sequential Innovation, Patents and Imitation" *RAND Journal of Economics*, forthcoming

Boldrin, Michele and David Levine (2009): A Model of Discovery, *American Economic Review Papers and Proceedings* **99**: 337-342

Kremer, Michael (1998): Patent Buyouts: a Mechanism for Encouraging Innovation, *Quarterly Journal of Economics* **93**: 1137–1167

Prasad, Ganesh (2001): Open Source-onomics: Examining some pseudo-economic arguments about Open Source `http://www.freeos.com/articles/4087`

Salop, Steven C. (1979): Monopolistic Competition with Outside Goods, *Bell Journal of Economics* **10**: 141-156

Tirole, Jean (1988): *Theory of Industrial Organization*, MIT Press

Summary. Considerable investments in Open Source software projects have been made in the last years by competing firms who could have free-ridden on the others' efforts. We identify two properties, one of decreasing returns to knowledge production and the other of preference for differentiated products in a monopolistically competitive market, which explain the phenomenon.

Keywords: R&D, Open Source, Free-Riding.

JEL classification: L1