KNOWLEDGE TRANSFER IN R&D OUTSOURCING: AN INCENTIVE-CONSTRAINED VIEW

SALVATORE MODICA*

Abstract. We study a dynamic incentive–constrained problem facing a firm wishing to outsource R&D activity. The problem is how to release information to the innovator over time, with concurring profit-sharing contract. Optimal policy involves knowledge transfer and innovator's profit share both increasing with time.

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1. Introduction

The practice of contracting out research about technological advancement of its product on the part of a firm develops during the nineties in response to the need to expand research capabilities in the face of increasing competitive pressure. As both Holmström–Roberts [6] and Kimsey–Kurokawa [7] report, one often observes strategically critical research being contracted out, as in the case of product design in automobile industry. Of this "something of a trend today toward disintegration, outsourcing, contracting out, and dealing through the market" ([6] p.80) one finds more circumscribed evidence in Thayer [14] and Birch [5] respectively for the chemical and the pharmaceutical industry, where the R&D outsourcing market in has grown at an average annual rate of 14.6% between 1997 and 2001. In other cases, for example in the case of development of a computer's operating system, research activity is usually pursued within the firm.

From a theoretical point of view the first paper dealing with the problem of research outsourcing is to our knowledge Aghion–Tirole [1], who study optimal allocation of property rights on innovation in a one–shot interaction between firm and innovator; they conclude that control should be allocated to the innnovator, in terms of the present paper that R&D outsourcing is more efficient than internal product development, if the innovator's effort is 'important enough' (ibidem p.1191). Assuming that this is the case, and also that there are no problems with ex-ante definability of the nature of innovation, we highlight a further potential obstacle to outsourcing, which is the following: to make the innovator productive, the firm has to transmit her some information about its existing technology and internal processes; but this information may be valuable to the innovator independently of her relationship with the firm, possibly so much that she might just walk away with it and default on her contractual obligations with the firm.

Knowledge management is a problem of increasing relevance already within the firm; Rajan–Zingales [11], who make it the main ingredient of their theory of the firm, report for example that 71% of the firms included in the Inc 500, a list made up of young firms, were founded by people who replicated or modified an idea encountered in their previous employment. Information leaking being a problem

* January 2006. Financial Support: MIUR. Affiliation: Dip. Scienze Economiche Finanziarie e Aziendali, Università di Palermo, Viale delle Scienze, 90128 Palermo, Italy. email modica@unipa.it.

within a firm, it can only get worse when it comes to transferring knowledge to third parties. In the extreme it can become a deadlock, when critical bits of information need to be revealed to give the outside innovator the chance to produce valuable results. The remedy is often to establish a long term relationship, a 'preferred-client relationship' in business language. [1] This is what the present paper studies, formalized as a multi-period principal-agent incentive constrained problem, where principal and agent are respectively firm and innovator. By and large, we find that the firm's optimal plan, when it is not the null contract, involves knowledge transfer and agent's profit share both increasing in time.

Optimal contract is the null contract in those cases where the amount of knowledge that the innovator needs to be productive is so large to make default more appealing than cooperation. This may be the case of the development of a computer's operating system: to work on it one has to know it all. From this perspective one may argue that the recent advancement of Linux, an operating system whose code is public domain, at the expenses of the privately owned Microsoft Windows, might be due to the fact that the Linux community, made up of several thousand developers scattered around the world, did not face the knowledge–transfer problem that prevented Microsoft from hiring all those smart programmers with ad hoc outsourcing contracts. [2]

Related Literature. The present paper studies optimal release of R&D-related information over a long horizon, a dynamical aspect of a problem which is usually modelled as a one-shot or two-stage game. Some papers, like Anton–Yao [2] or the more recent Baccara–Razin [3], address the problem which an inventor who has *already* produced an idea faces of how to approach others who may help him to translate the idea into a produced good. Others, like Rajan–Zingales [11] and Zábojník [15], analyse information protection and circulation in a hierarchical structure *within* a firm. Explicitly dealing with R&D outsourcing is a paper by Lai, Riezman and Wang [9] who consider a two-dimensional contract between firm and innovator specifying a fixed payment to the latter besides a profit-sharing rule. The informational constraint on the firm is more gentle than in our case, in the sense that information protection is not vital to the firm, and in fact leakage may occur (and be common knowledge) in the equilibrium relationship; the authors examine in a comparative static framework various factors which influence the decision on the part of the firm about whether conducting R&D in-house or via outsourcing.

From the mathematical point of view the problem we analyze is one of finding, in Ray [12] terminology, a sequence of Self–Enforcing Agreements. Compared to the class of problems studied by Ray we have more special forms of agreements, but on the other hand impose no stationarity on the functions involved. Ray identifies a general qualitative property of the efficient feasible sequences in this type of problems, namely that after some time the continuation sequence will maximize the agent's payoff over all efficient sequences. We explicitly solve the principal's problem for a class of cases, and in some of them find counterexamples to the above property (which is consistent with Ray's theorem given the non–stationarity of the environment).

The problem we study is stated in the next section and analysed in the following two. Section 5 summarizes, and an appendix contains proofs.

[1] See Birch [5], reporting on the success of this strategy in the pharmaceutical industry in the U.S. On the long–term nature of R&D outsourcing relationships in the U.S. and Japan see also Kimsey–Kurokawa [7] and Holmström–Roberts [6].

[2] Some notes and related material on the Linux story are contained for ease of reference in an appendix to this paper, available at `www.unipa.it/modica`.

2. Statement of the Problem

There are two actors in the model, a firm and an innovator. The innovator can improve her performance if she has some knowledge of the firm's internal processes. Abstracting altogether from uncertainty, we assume there is an exogenous process of knowledge base in the hands of the firm, $\{K_t\}_{t=0,1...}$, $K_t \in \mathbb{R}_+$, increasing and bounded. At time t, if the firm transfers knowledge $k_t \in [0, K_t]$ to the innovator, net benefit from the latter's work is $V_t(k_t)$; on the other hand the innovator has also the option of just walking away with k_t instead of working for the firm, and if he does this he gets a default value $D_t(k_t)$. The firm proposes a long term contract to the inovator, by choosing for each t a knowledge transfer k_t and a sharing rule $(\phi_t, 1-\phi_t)$, where ϕ_t and $1-\phi_t$ are its own and the innovator's shares of V_t. The firm's problem we formally analyse is then the following (with β discount factor):

$$\max_{\{k_t,\phi_t\}_{t\geq 0}} \sum_{t\geq 0} \beta^t\, \phi_t V_t(k_t)$$

subject to, for all t,

$$\begin{aligned} 0 \leq k_t \leq K_t, \quad k_t \leq k_{t+1} \quad \phi_t \leq 1, \\ \sum_{s\geq t} \beta^{s\text{-}t}(1-\phi_s)V_s(k_s) \geq D_t(k_t)\,, \quad \text{and} \\ \sum_{s\geq t} \beta^{s\text{-}t}\, \phi_s V_s(k_s) \geq 0. \end{aligned} \tag{P}$$

Besides the self-explaining constraints on the k_t sequence, the last two constraints are the innovator's incentive constraint and the firm's participation constraint, for discussion of which the reader may consult Ray [12]. It is imposed $\phi_t \leq 1$ but not $\phi_t \geq 0$ because we imagine the innovator to be liquidity constrained, but not the firm. The structural assumptions are that, for all t, V_t is increasing concave, D_t is increasing convex, and $V_t(0) = D_t(0) = 0$; we shall also assume that the default value does not grow too fast: $\beta D_{t+1} \leq D_t$. Since the sequence $\{K_t\}$ is bounded the problem is set up in ℓ_∞ (and its dual; details about this are in Appendix).

We are well aware that we are studying a much simplified problem. We sidestep in particular the issues addressed by Aghion–Tirole [1] of contractibility of effort and ex–ante definability of innnovation, which in a dynamic setting become highly relevant hold–up problems (How can the firm walk away if it suspects that the innovator's effort is too low, with strategic knowledge already in the latter's hands? See Kultti–Takalo [8] for a concrete instance of this); and the problem of observability of costs and values. Also, equally important is uncertainty in this context, in theory as well as in practice: uncertainty about research output given k, and uncertainty about the quality of the innovator.

Problem (P) above has the same structure of the one studied by Ray [12]; as we said in the introduction we have more special strategy sets (forms of agreements), but on the other hand impose no stationarity on the functions involved; and, while Ray identifies a general qualitative property of the efficient feasible agreement sequences in this type of problems, we explicitly solve the one in hand (when the feasible set is non-empty) for a class of cases. In the sequel we may call the two actors principal and agent.

3. No–contract Feasible set, and Linux–vs–Windows

The feasible set in problem (P) is never empty, because it contains all sequences $\{\phi_t, k_t\}$ with $k_t = 0$ all t (and $\phi_t \in [0, 1]$); if it contains no other points, the optimal contract between firm and innovator is the null contract. The conditions under which this occurs are easily spelled out. The extreme case is with $V_t \equiv 0$ $\forall t > 0$

and $V_0 < D_0$, and the general case is then clear: the innovator's productivity falls rapidly with time, and the first–shot outcome is not as valuable as the knowledge needed to produce it. This will be the case for example if the innovator's productivity is low for all but near–full knowledge transfer, and for such transfers on the other hand the default value is very high.

As mentioned in the introduction, it is our contention that the above scenario fits well the Linux–v–Windows story. Indeed, to develop an operating system a programmer cannot do without knowing it deeply. Moreover it is often by inspecting its various aspects that she finds the one whose improvement best fits her capabilities —as in science. Finally, and the parallel to science is again inevitable, often a programmer's best shots are his first ones: in our notation, V_t decreases sharply with time. The other part of the argument is that the magnitude of the default value D_0 may well be high compared to V_0 for near-full knowledge transfer, and the rationale for this is that the value of *critical* knowledge may be in the order of the value of the firm itself; if this is the case, the order of magnitude of $D_0(K_0)$ is the discounted sum of all future $V_t(K_t)$'s; in particular, it seems reasonable to think that the value of the source code of the Windows operating system is uncomparably higher than the potential value of a developer's contribution.

So an explanation of the technical explosion of the Linux operating system with little reaction on the part of the incumbent Microsoft monopolist is that the latter, owing to knowledge–transfer problems, was forced to hang on to relatively few in-house programmers, and was therefore totally unfit to compete with the army of open–source developers and testers who contributed, with no problems of that sort, to make Linux the powerful operating system which it is today. We stress that there was no knowledge problem on the Linux side, the reason being simply that knowledge there is common property (cfr. e.g. the appendix cited in footnote 2).

4. Non–trivial Feasible Set: Optimal Outsourcing

We now assume that the zero knowledge–transfer path is dominated by other feasible alternatives, and begin to analyse the problem by writing the Lagrangean and imposing stationarity and complementary slackness conditions. Justification of the procedure is in Appendix.

Inspection of problem (P) page 3 reveals that principal and agent have here a common interest —that V be as high as possible; thus intuitively the solution should call for as large a transfer of k as it is compatible with the agent's incentive constraint, and this is what formal analysis confirms. As for profit sharing, it is in the spirit of Ray's result that the principal should appropriate her total payoff in finite time; we will find that this is indeed what happens when the agent's default value is 'low' relatively to the value of his research contribution, in which case in fact the principal's share is zero from some point on. On the contrary, when the default option is more attractive the optimal contract calls for positive shares for both players at all times. We turn now to formal analysis.

The Lagrangean of problem (P) without the non-negativity and increasingness constraints on k_t (which will be satisfied in all cases we consider) is

$$\mathcal{L} = \sum_{t\geq 0} \beta^t \Big[\phi_t V_t(k_t) + \xi_t(K_t - k_t) + \zeta_t(1-\phi_t)\Big]$$
$$+ \sum_{s\geq 0} \lambda_s \Big[\sum_{t\geq s} \beta^{t\text{-}s}(1-\phi_t)V_t(k_t) - D_s(k_s)\Big] + \sum_{s\geq 0} \mu_s \Big[\sum_{t\geq s} \beta^{t\text{-}s}\, \phi_t V_t(k_t)\Big].$$

Letting

$$\nu_t = \lambda_0\beta^t + \lambda_1\beta^{t-1} + \cdots + \lambda_t, \quad \rho_t = \mu_0\beta^t + \mu_1\beta^{t-1} + \cdots + \mu_t$$

this becomes

$$\mathcal{L} = \sum\nolimits_{t\geq 0} \beta^t \Big[\xi_t(K_t - k_t) + \zeta_t(1-\phi_t) - \beta^{-t}\lambda_t D_t(k_t)\Big] + \sum\nolimits_{t\geq 0} V_t(k_t)\Big[(\beta^t + \rho_t)\phi_t + \nu_t(1-\phi_t)\Big]\,.$$

Thus complementary slackness and FOC are

$$\xi_t(K_t - k_t) = 0, \quad \zeta_t(1-\phi_t) = 0 \;\; \forall t,$$
$$\lambda_s\Big[\sum\nolimits_{t\geq s} \beta^{t\text{-}s}(1-\phi_t)V_t(k_t) - D_s(k_s)\Big] = 0, \quad \mu_s \sum\nolimits_{t\geq s} \beta^{t\text{-}s}\,\phi_t V_t(k_t) = 0 \;\; \forall s,$$
$$\big[(\beta^t + \rho_t)\phi_t + \nu_t(1-\phi_t)\big]\, V_t'(k_t) - \lambda_t D_t'(k_t) = \beta^t \xi_t\,,$$
$$\big[\beta^t + \rho_t - \nu_t\big]\, V_t(k_t) = \beta^t \zeta_t \;\; \forall t\,.$$

So the FOC with respect to k_t is

$$\begin{aligned} \big[(\beta^t + \rho_t)\phi_t + \nu_t(1-\phi_t)\big]\, V_t'(k_t) - \lambda_t D_t'(k_t) &\geq 0, \\ &= 0 \text{ if } k_t < K_t\,. \end{aligned} \tag{1}$$

And given $V_t(k_t) > 0\ \forall k_t > 0$, the one with respect to ϕ_t is $\beta^t + \rho_t - \nu_t \geq 0$, equal if $\phi_t < 1$; this is more conveniently rewritten as

$$\begin{aligned} (\lambda_0 - \mu_0) + (\lambda_1 - \mu_1)\beta^{-1} + \cdots + (\lambda_t - \mu_t)\beta^{-t} &\leq 1, \\ &= 1 \text{ if } \phi_t < 1\,. \end{aligned} \tag{2}$$

Now observe that $\phi_t = 1\ \forall t$ is not feasible, for it violates the agent's incentive constraint (we are dealing with solutions with non-zero $\{k_t\}$, for which the V_t and D_t functions are non-zero). Let t_0 be the first t such that $\phi_t < 1$. Then from (2), after t_0 the first s such that $\lambda_s \neq \mu_s$ should have $\lambda_s < \mu_s$. This would imply that inequality in (2) strict at s, whence $\phi_s = 1$; it would also imply $\mu_s > 0$, which by complementary slackness gives $\sum_{t\geq s} \beta^{t\text{-}s}\,\phi_t V_t(k_t) = 0$; and the latter, with $\phi_s = 1$, would then imply that the principal participation constraint is violated at $s+1$. Conclusion: after t_0 one has $\lambda_t = \mu_t\ \forall t$. Therefore,

for all $t > t_0$: either (i) $\lambda_t = \mu_t = 0$; or (ii) $\lambda_t = \mu_t > 0$.

In case (i), $\nu_t - \rho_t = \beta^t$ (for all $t > t_0$): indeed at t_0, since $\phi_{t_0} < 1$, equation (2) says $\nu_{t_0} - \rho_{t_0} = \beta^t$, so $\nu_{t_0+1} - \rho_{t_0+1} = \beta(\nu_{t_0} - \rho_{t_0}) + (\lambda_{t_0+1} - \mu_{t_0+1}) = \beta^{t_0+1}$, etc. Thus (1) becomes $(\beta^t + \rho_t)V_t'(k_t) \geq 0$, equal if $k_t < K_t$. But both factors on the left side of the inequality are positive, so $k_t < K_t$ cannot be. Thus in case (i) $k_t = K_t\ \forall t > t_0$.

In case (ii), from the complementary slackness conditions on λ and μ we get, for each $s > t_0$,

$$\sum\nolimits_{t\geq s} \beta^{t\text{-}s}\,\phi_t V_t(k_t) = 0 \;\text{ and }\; \sum\nolimits_{t\geq s} \beta^{t\text{-}s}\, V_t(k_t) = D_s(k_s)\,.$$

The first set of equalities imply that $\phi_t = 0\ \forall t > t_0$; so after t_0 all value goes to the agent, and $\{k_t\}_{t>t_0}$ is chosen so that this value just covers default value D_t at each t (recalling that V_t are concave and D_t convex, this means that he chosen k's could not be higher without violating the agent's incentive constraint). In this case the structure of the solution is the same as that found by Ray in [12] in the stationary case.

In both cases, the picture for $t < t_0$ is that since $\phi_t = 1$, the principal's individual rationality constraints are met with strict inequality, whence $\mu_t = 0$; so (1) becomes

$$\begin{aligned} \beta^t V_t'(k_t) &\geq \lambda_t D_t'(k_t), \\ &= \qquad\qquad \text{if } k_t < K_t. \end{aligned}$$

We next explicitly describe the solution in three special cases, which will differ in the optimal amount of knowledge which can be transferred in equilibrium. Letting $\mathcal{V}_t$ denote the highest value the innovator could possibly realize from time t onwards, i.e.

$$\mathcal{V}_t = \sum\nolimits_{s \geq t} \beta^{s-t} V_s(K_s)\,,$$

we consider the situations where one of the following conditions holds *for all* t:

$$D_t(K_t) = \mathcal{V}_t; \quad D_t(K_t) > \mathcal{V}_t; \quad D_t(K_t) < \mathcal{V}_t.$$

The last is a case where innovating has more value than defaulting, so intuitively the solution should have $k_t = K_t$ all t; we shall confirm formally that this is so, except possibly for a finite number of initial periods. In the second, opposite case such full knowledge transfer is clearly not feasible: the feasible set can be 'thin', so the first step is to impose conditions which guarantee that it is large enough for the problem to be interesting; here stationarity seems to be the single most natural assumtption to make. The first case is obviously a measure–zero set, but it is instructive for the solutions of the others, and with it we start.

Case $D_t(K_t) = \mathcal{V}_t\ \forall t$**.** We shall see that this case falls under heading (ii) above. We make here *two assumptions*. First, in keeping with the spirit of the present section that no–knowledge–transfer is dominated, we assume that $V_0'(0) > D_0'(0)$; since $D_0(0) = V_0(0) = 0$ and $D_0(K_0) = V_0(K_0) + \beta\mathcal{V}_1 > V_0(K_0)$, we then have

$$0 < \text{argmax}_{k_0}\, V_0 - D_0 < K_0. \tag{3}$$

The second is a technical assumption, in the spirit of the ℓ_∞ setup:

$$\textit{The sequence } (\mathcal{V}_t)_{t \geq 0} \textit{ is bounded.} \tag{4}$$

In the next proposition optimal policy is characterized. The idea is that the principal wants to push knowledge transfer k_t up to K_t as soon as she can and leave all value to the agent from then on (feasibility and $k_t = K_t$ implies $\phi_t = 0$), conditional on being able to appropriate the value coming out of the initial phase. The optimal policy would be to do this in period 1 if the agent were not liquidity constrained (that is if there were no constraint $\phi_0 \leq 1$); with this constraint the initial phase may last longer.

To get some intuition we may start by observing that the agent's incentive constraint at $t = 0$ is just

$$\sum\nolimits_{t \geq 0} \beta^t\, \phi_t V_t(k_t) \leq \sum\nolimits_{t \geq 0} \beta^t\, V_t(k_t) - D_0(k_0)\,; \tag{5}$$

so if the right member is maximized subject to the other constraints and to being equal to the left member (which is the principal's payoff), the solution to (P) is found. To see how the optimization process goes observe that the highest value the right side of (5) can take is $[\max_{k_0} V_0 - D_0] + \beta\mathcal{V}_1$; and that setting $k_t = K_t\ \forall t \geq 1$ forces $\phi_t = 0\ \forall t \geq 1$ by the assumption $D_t(K_t) = \mathcal{V}_t$; the latter also implies that the sequence $\phi_t = 0, k_t = K_t\ \forall t \geq 1$ satisfies all incentive constraints for such t's. Therefore if there is a $\phi_0 \leq 1$ such that the incentive constraint at $t = 0$ holds with equality, that is such that

$$\phi_0 V_0(k_0) = V_0(k_0) - D_0(k_0) + \beta\mathcal{V}_1 \quad \text{with} \quad k_0 = \text{argmax}(V_0 - D_0),$$

then (5) holds with equality and the problem is solved: $k_0 = \text{argmax}(V_0 - D_0)$, $k_t = K_t\ \forall t \geq 1$, ϕ_0 defined by the last equation displayed, and $\phi_t = 0\ \forall t \geq 1$. In this case the principal obtains all of her payoff in the first period. Problem is that a $\phi_0 \leq 1$ as required above may not exist, that is, it may be that

$$D_0(k_0) \geq \beta\mathcal{V}_1, \quad k_0 = \text{argmax}(V_0 - D_0) \tag{6}$$

fails. In the latter case the next step is to try and maximize the right member of (5) with respect to k_0 and k_1, leaving $k_t = K_t$ $\forall t \geq 2$, while respecting the agent's incentive constraints and subject to finding ϕ_0 and ϕ_1 ($\phi_t = 0$ $\forall t \geq 2$) such that at the maximizing values of k_0, k_1 relation (5) hold with equality. this problem, optimal $\phi_0 = 1$; so if the optimal *unconstrained* value of ϕ_1 is ≤ 1 then again we have found the solution of (P), and the principal gets all her payoff in the first two periods. We shall show in appendix that the process stops in a finite number of stages, and this leads to the proposition which follows.

As to the statement below, observe that given $D_t(K_t) = \mathcal{V}_t$, problem (P) reduces to the one appearing there when after t_0 one imposes $\phi_t = 0$ and $k_t = K_t$ (and neglects $\phi_{t_0} \leq 1$). Again, proof is in appendix.

Proposition 1. *In the case $D_t(K_t) = \mathcal{V}_t$ $\forall t$, assuming $V_0'(0) > D_0'(0)$ and that the sequence $(\mathcal{V}_t)_{t\geq 0}$ is bounded, there exists a first time $t_0 < \infty$ such that the problem*

$$\max_{\{k_t,\phi_t\}_{t=0,\ldots,t_0}} \sum_{t=0}^{t_0} \beta^t V_t(k_t) + \beta^{t_0+1}\mathcal{V}_{t_0+1} - D_0(k_0)$$

subject to

$$k_t \leq K_t,\ t = 0,\ldots,t_0 \quad \phi_t \leq 1,\ t = 0,\ldots,t_0 - 1$$

$$\sum_{s=t}^{t_0} \beta^{s-t}\phi_s V_s(k_s) = \sum_{s=t}^{t_0} \beta^{s-t} V_s(k_s) + \beta^{t_0+1-t}\mathcal{V}_{t_0+1} - D_t(k_t),\ t = 0,\ldots,t_0$$

has optimal $\phi_{t_0} \leq 1$ (from the last of the above constraints, this ϕ_{t_0} is defined by the equation $\phi_{t_0}V_{t_0}(k_{t_0}) = V_{t_0}(k_{t_0}) + \beta\mathcal{V}_{t_0+1} - D_{t_0}(k_{t_0})$). The problem has optimal $\phi_t = 1$ for all $t < t_0$.

Optimal policy for problem (P) is given by the solution to the above problem, followed by $\phi_t = 0$ and $k_t = K_t$ for all $t > t_0$.

Remark. It is shown in the proof that for all $t < t_0$ the optimal policy has $D_t(k_t) = \beta D_{t+1}(k_{t+1})$; thus the optimal amount of knowledge transfer is increasing for all t (using the assumption $\beta D_{t+1} \leq D_t$ for the initial phase).

Case $D_t(K_t) > \mathcal{V}_t$ $\forall t$. This case also falls under heading (ii) of page 5. In this case the vaule of the default option is relatively high, and so it is the closest to the no–contract feasible set. To guarantee that a non–trivial set of contracts is feasible, as we said before we find that imposing stationarity is the single more natural assumption. We then *assume* here that:

$$V_t = V, \quad D_t = D, \quad K_t = K, \quad \forall t \geq 0. \tag{7}$$

Thus the inequality $D_t(K_t) > \mathcal{V}_t$ $\forall t$ becomes

$$D(K) > (1-\beta)^{-1}V(K).$$

Given this, $V(0) = D(0) = 0$ and concavity of $V - (1-\beta)D$, if the derivative at zero of the latter function is non–positive the only stationary sustainable level of knowledge transfer is zero even with $\phi = 0$, because it would be $D(k) > (1-\beta)^{-1}V(k)$ $\forall k > 0$. We then *also assume* that $V'(0) > (1-\beta)D'(0)$ (this is weaker than the corresponding assumption in the previous case). Hence there exist a largest level of stationary sustainable knowledge transfer $0 < k^* < K$, defined by

$$D(k^*) = (1-\beta)^{-1}V(k^*). \tag{8}$$

As in the previous case the idea is that the principal wants to push up k_t as soon as she can; but now not up to K_t, which is unfeasible, but up to k^*.

Proposition 2. *In the case* $D_t(K_t) > \mathcal{V}_t \ \forall t$*, assuming stationarity* (7) *and that* $V'(0) > (1-\beta)D'(0)$*, all of Proposition 1 can be restated, with the following two modifications:* $\mathcal{V}_{t_0+1}$ *in the problem there described replaced by* $(1-\beta)^{-1}V(k^*)$*, and the final statement "*$k_t = K_t$ *for all* $t > t_0$*" changed to "*$k_t = k^*$ *for all* $t > t_0$*".*

Case $D_t(K_t) < \mathcal{V}_t \ \forall t$**.** This case will fall under heading (i) of page 5. As we already observed one would guess that full knowldege transfer is optimal; this is only partially true, because as we shall see it may happen that full transfer does not begin at time zero. To ease exposition we shall again make two simplifying assumptions. The first is in the spirit of stationarity:

$$\mathcal{V}_t = \mathcal{V} \ \forall t \geq 0. \tag{9}$$

This is equivalent to assuming $V_t(K_t) = V_0(K_0) \ \forall t \geq 0$ ((9) is obviously implied by the latter; but given (9) one has $\mathcal{V}_t = V_t(K_t) + \beta\mathcal{V}_{t+1} = V_t(K_t) + \beta\mathcal{V}_t$ whence $V_t(K_t) = (1-\beta)\mathcal{V}_t = (1-\beta)\mathcal{V} \ \forall t$). Next, although default value is 'small' here, we still find it natural to imagine that the gap between $\mathcal{V}$ and $D_t(K_t)$ would shrink with time. We assume that this occurs at a constant rate, in the sense that for some $\gamma \in (0,1)$ one has

$$\mathcal{V}_t - D_t(K_t) = \gamma\big[\mathcal{V}_{t-1} - D_{t-1}(K_{t-1})\big], \quad t > 0, \tag{10}$$

which given (9) is obviously equivalent to $D_t(K_t) = (1-\gamma)\mathcal{V} + \gamma D_{t-1}(K_{t-1})$.

Now define

$$\hat{\phi}_t = \frac{1-\beta\gamma}{1-\beta}\,\frac{\mathcal{V} - D_t(K_t)}{\mathcal{V}}\,.$$

Since $\mathcal{V} - D_t(K_t) = \gamma^t\big[\mathcal{V} - D_0(K_0)\big]$, clearly $\hat{\phi}_t = \gamma^t\hat{\phi}_0$. Call $t_0 \geq 0$ the first t such that $\hat{\phi}_t \leq 1$. Optimal policy is then as follows (proof in appendix):

Proposition 3. *In the case* $D_t(K_t) < \mathcal{V}_t \ \forall t$*, under assumptions* (9) *and* (10)*, for the* $t_0 \geq 0$ *just defined, optimal policy has* $k_t = K_t$ *and* $\phi_t = \hat{\phi}_t \ \forall t > t_0$*. Optimal* k_t *and* ϕ_t *for* $t \leq t_0$ *are specified in the appendix.*

For example, if $\hat{\phi}_{t_0} = 1$ the policy for this initial phase is $k_t = K_t, \ \phi_t = 1 \ \forall t \leq t_0$; otherwise the latter is usually not feasible (again details in appendix).

Notice that in the present case the principal's continuation payoff, although decreasing to zero, remains positive forever.

5. Conclusion

R&D outsourcing is a potentially unlimited source of expansion of a firm's research activity. Of course its implementation is limited by the difficulty of writing appropriate contracts, because of the uncertainty surrounding the nature of research output and its true value and costs, and of the limited information about the research capability of the potential innovator. Considering situations where all this is overcome, we highlight a further problem that a firm outsourcing R&D must deal with, namely how to release information about its internal processes to the research firm over time, trading off the innovator's productivity against the risk of her default (both increasing with knowledge transfer). The general result emerging from analysis is that the optimal long term contract involves knowledge–transfer and innovator's share of profits both increasing with time; so in equilibrium the firm appropriates its total payoff in early stages while the innovator keeps working in view of higher profits in the more distant future.

Appendix: Mathematical Arguments

Justifying the Lagrangean. There are two steps involved in writing the Lagrangean the way we have done (i.e. the 'usual' way) and imposing its stationarity in this context. The first concerns existence of multipliers in the dual of ℓ_∞; the second regards conditions ensuring that those multipliers are in fact in ℓ_1 (a subset of the above dual), i.e. expressible as a sequence of real numbers. For both we shall merely invoke existing theorems.

Existence of multipliers in the positive cone of ℓ^*_∞ such that at the optimal solution the lagrangean is stationary follows for instance from theorem 1.10 of chapter 3 (p.190) of Barbu–Precupanu [4]. The regularity condition (ib. p.191) in our case amounts to the requirement that the inequality hold for s sufficiently large along the optimal sequence. Conditions ensuring this are easy to write in all cases considered in the paper (a simple, but unappealing one is that β be close enough to one).

As to the ℓ_1 problem, we can apply corollary 5.6 of Rustichini [13] directly; validity of its hypotheses in our case is immediate to check.

Proof of Proposition 1. We resume the argument where it was interrupted, after (6). As we were saying, if that relation fails one turns to the two–period problem, which is the following:

$$\max_{(\phi_0,\phi_1),(k_0,k_1)} V_0 + \beta V_1 + \beta^2 \mathcal{V}_2 - D_0$$

subject to

$$\phi_0 \le 1, \quad k_t \le K_t,\ t = 0,1 \qquad (P_2)$$

$$\phi_0 V_0 + \beta\phi_1 V_1 = V_0 + \beta(V_1 + \beta\mathcal{V}_2) - D_0 \qquad (a)$$

$$\phi_1 V_1 = V_1 + \beta\mathcal{V}_2 - D_1. \qquad (b)$$

Substituting the last constraint the problem can be written as

$$\max_{(k_0,k_1)} V_0 + \beta\left[V_1 + \beta\mathcal{V}_2\right] - D_0$$

$$\text{subject to} \quad D_0 \ge \beta D_1, \quad k_t \le K_t,\ t+0,1. \qquad (P_2')$$

Suppose that the solution to this problem has $D_1 \ge \beta\mathcal{V}_2$; then the value of ϕ_1 defined by (b) satisfies $\phi_1 \le 1$, so that (ϕ_0,ϕ_1) defined by (a) and (b) together with the solution (k_0,k_1) of (P_2') solve (P): to wit, the solution of the latter is $\phi_0 = 1$ (which follows from the fact that the constraint $D_0 \ge \beta D_1$ in (P_2') is binding, as will be verified shortly), ϕ_1 defined by (b), (k_0,k_1) solving (P_2'), and $\phi_t = 0$, $k_t = K_t$ $\forall t \ge 2$. In this case the principal gets her payoff in the first two periods, and from then on only the agent's payoff is positive.

If the solution to (P_2') has $D_1 < \beta\mathcal{V}_2$, then we pass to the obvious next step, which is the three–period try. We shall show that this process ends in a finite number of steps, but before moving on we must check that the constraint $D_0 \ge \beta D_1$ is binding in (P_2'). The lagrangean is

$$V_0 - D_0 + \beta\left[V_1 + \beta\mathcal{V}_2\right] + \lambda(D_0 - \beta D_1) + \mu_0(K_0 - k_0) + \beta\mu_1(K_1 - k_1);$$

so FOC and complementary slackness give

$$V_0' - (1-\lambda)D_0' \ge 0, \quad = 0 \text{ if } k_0 < K_0, \qquad (F_2\, i)$$

$$V_1' - \lambda D_1' \ge 0, \quad = 0 \text{ if } k_1 < K_1. \qquad (F_2\, ii)$$

If $k_1 < K_1$ then from $(F_2\, ii)$ $\lambda = V_1'/D_1' > 0$, so the constraint in question binds. Thus it may be non–binding only if $k_1 = K_1$; with this and $\lambda = 0$, $(F_2\, i)$ reads

$V_0' - D_0' \geq 0,\ = 0$ if $k_0 < K_0$. But since the max of the concave function $V_0 - D_0$ is interior by (3) one has $V_0' - D_0' < 0$ at K_0, so it cannot be $k_0 = K_0$; so it should be $k_0 < K_0$; but then $V_0' - D_0' = 0$, i.e. $k_0 = \text{argmax}(V_0 - D_0)$; on the other hand, for this pair (k_0, K_1) we have by failure of (6) $D_0 < \beta\mathcal{V}_1 = \beta D_1$, i.e. the pair is not feasible for the problem in hand. We conclude that the constraint is binding, and so optimal $\phi_0 = 1$ in (P_2). We observe for future reference that it has also been shown that $\lambda > 0$; this implies, via $(F_2\, i)$, that $k_0 > \text{argmax}(V_0 - D_0) > 0$.

To see what is involved in showing that the the process ends in a finite number of steps let us look again at the inequality $D_1 \geq \beta\mathcal{V}_2$ in (P_2'); since we have just found $D_0 = \beta D_1$ at the optimum, this is

$$D_0(k_0) \geq \beta^2\mathcal{V}_2, \quad k_0 \text{ solving } (P_2'). \tag{11}$$

Comparing this with (6) we guess that the s-period try will be the successful one if the inequality $D_0(k_0) \geq \beta^s\mathcal{V}_s$ holds for k_0 optimal solution of the relevant problem. Since it will be shown that this k_0 will always be not smaller than $\text{argmax}\,(V_0 - D_0)$, by the boundedness assumption (4) the inequality will be satisfied for s large enough.

We turn to the $(s+1)$–period problem, in the variables $k_0, \ldots, k_s$. The hypothesis is that for the sequence $k_0, \ldots, k_{s-1}$ solving the s–period problem, one has $D_{s-1} < \beta\mathcal{V}_s$; and that similarly for $k_0, \ldots, k_{s-2}$ solving the $(s-1)$–period problem one has $D_{s-2} < \beta\mathcal{V}_{s-1}$; and so on down to the one–period problem. In words, the induction hypothesis is that problem (P) cannot be solved by the principal appropriating all of her payoff in less than $s+1$ periods.

We consider the $(s+1)$–period analogue of problem (P_2) and arrive at the $(s+1)$–period version of problem (P_2'), which is, omitting the constant term $\beta^{s+1}\mathcal{V}_{s+1}$,

$$\max_{(k_0,\ldots,k_s)} \sum_{t=0}^{s} \beta^t V_t - D_0$$

$$\text{subject to} \tag{P_s'}$$

$$k_t \leq K_t,\ t = 0, \ldots, s$$
$$D_t \geq \beta D_{t+1},\ t = 0, \ldots, s-1.$$

Again our aim is to show that the constraints on D are binding. For then the question whether $\phi_s \leq 1$, i.e. $D_s \geq \beta\mathcal{V}_{s+1}$, becomes $D_0 \geq \beta^{s+1}\mathcal{V}_{s+1}$. The lagrangean for (P_s') is

$$V_0 - D_0 + \lambda_0(D_0 - \beta D_1) + \mu_0(K_0 - k_0) + \beta\left[V_1 + \lambda_1(D_1 - \beta D_2) + \mu_1(K_1 - k_1)\right] + \ldots$$
$$+\beta^{s-1}\left[V_{s-1} + \lambda_{s-1}(D_{s-1} - \beta D_s) + \mu_{s-1}(K_{s-1} - k_{s-1})\right] + \beta^s\left[V_s + \mu_s(K_s - k_s)\right].$$

Stationarity and complementary slackness give

$$V_0' - (1 - \lambda_0)D_0' \geq 0, \quad = 0 \text{ if } k_0 < K_0$$
$$V_1' - (\lambda_0 - \lambda_1)D_1' \geq 0, \quad = 0 \text{ if } k_1 < K_1$$
$$\cdots\cdots\cdots$$
$$V_{s-1}' - (\lambda_{s-2} - \lambda_{s-1})D_{s-1}' \geq 0, \quad = 0 \text{ if } k_{s-1} < K_{s-1}$$
$$V_s' - \lambda_{s-1}D_s' \geq 0, \quad = 0 \text{ if } k_s < K_s.$$

As in the two–period case, from the last condition displayed we deduce that for $D_{s-1} - \beta D_s$ not to be binding it must be $k_s = K_s$, and $\lambda_{s-1} = 0$. But then the rest of the conditions are exactly the same as those of the s–period problem, in which case the solution would be the same as that of the s–period problem followed by $k_s = K_s$; but then the hypothesis implies that $D_{s-1} < \beta\mathcal{V}_s = \beta D_s$, contradicting feasibility; so $D_{s-1} = \beta D_s$.

Next $D_{s-2}-\beta D_{s-1}$. If $k_{s-1} < K_{s-1}$ then as before $\lambda_{s-2}-\lambda_{s-1} = V'_{s-1}/D_{s-1} > 0$ which would imply that the constraint is binding. If on the other hand $k_{s-1} = K_{s-1}$ and $\lambda_{s-2} = 0$, we are back to the $(s-1)$–period problem, which with $k_{s-1} = K_{s-1}$ has $D_{s-2} < \beta D_{s-1}$, contradicting feasibility again. So $D_{s-2} - \beta D_{s-1}$ is binding, and continuing this way we conclude that all the D constraints are binding. It has also been shown, incidentally, that always $\lambda_0 > 0$.

Now, as anticipated, given $D_0 = \beta D_1 = \cdots = \beta^s D_s$ (and $\phi_0 = \cdots = \phi_{s-1} = 1$ in problem (P_s)), the question whether $\phi_s \leq 1$, i.e. $D_s \geq \beta \mathcal{V}_{s+1}$, becomes

$$D_0(k_0) \geq \beta^{s+1}\mathcal{V}_{s+1}, \tag{12}$$

k_0 being part of the solution to the s–period problem. And this holds for s sufficiently large. Indeed, in any s–period problem either $k_0 = K_0$, or from complementary slackness $V'_0 - D'_0 = -\lambda_0 D'_0 < 0$, last inequality from $\lambda_0 > 0$; thus in all problems the optimal $k_0 > \operatorname{argmax}(V_0 - D_0)$, whence the left member of (12) is bounded away from zero; on the other hand, by assumption (4) the right member tends to zero as s diverges. This concludes the argument.

Proof of Proposition 2. As we did in the previous case we rewrite the agent's incentive constraints, the first one binding:

$$\sum\nolimits_{t\geq 0} \beta^t\, \phi_t V(k_t) = [V(k_0) - D(k_0)] + \sum\nolimits_{t\geq 1} \beta^t V(k_t) \tag{13}$$

$$\sum\nolimits_{t\geq s} \beta^{t\text{-}s}\, \phi_t V(k_t) \leq \sum\nolimits_{t\geq s} \beta^{t\text{-}s}\, V(k_t) - D(k_s),\ \ s \geq 1\,. \tag{14}$$

Forget as before the constraint $\phi_0 \leq 1$. In the previous case it was then immediate that the max the right member of (13) was $[\max_{k_0} V_0 - D_0] + \beta\mathcal{V}_1$, and that this choice of $\{k_t\}_{t\geq 0}$ satisfied (with equality) the other incentive constraints. In the present case the situation is slightly more complex: the choice $k_t = K\ \forall t \geq 1$ is unfeasible, and then maximization of $\sum_{t\geq 1} \beta^t V(k_t)$ with respect to $\{k_t, \phi_t\}_{t\geq 1}$ subject to the constraints (14) is non–trivial. We shall now show that it is solved by $k_t = k^*$, $\phi_t = 0\ \forall t \geq 1$. Thus if for this choice (with $k_0 = \operatorname{argmax}(V - D)$) the ϕ_0 defined by $\phi_0 V(k_0) = V(k_0) - D(k_0) + \beta(1-\beta)^{-1}V(k^*)$ happens to be ≤ 1, problem (P) is solved.

If not, as before the principal has to try and appropriate his payoff in two periods. In this case again the difference compared to the case $D_t(K_t) = \mathcal{V}_t\ \forall t$ is that we have a non–trivial maximization, of $\sum_{t\geq 2} \beta^t V(k_t)$ under the constraints in (14) for $s \geq 2$; but again it is be proved by the same argument as the one we are about to give that the solution to this maximum problem is $k_t = k^*, \phi_t = 0\ \forall t \geq 2$. Thus at stage two we are again in a position analogous to that of case $D_t(K_t) = \mathcal{V}_t\ \forall t$, with $\mathcal{V}_2$ replaced by $(1-\beta)^{-1}V(k^*)$ in problem (P_2) of page 9. At this point the argument parallels the previous one: optimal $\phi_0 = 1$, and if the ϕ_1 defined by the equation $\phi_1 V(k_1) = V(k_1) - D(k_1) + \beta(1-\beta)^{-1}V(k^*)$, with k_1 part of the solution of the modified (P_2), is ≤ 1, then problem (P) is solved. Otherwise one goes to stage three, etc. until payoff appropriation is possible. The concluding part of the argument is as before.

It is thus left to analyse maximization $\sum_{t\geq 1} \beta^t V(k_t)$ over the set defined by (14). We shall show that the sequence $k_t = k^*\ \forall t \geq 1$ maximizes the given function on a larger set, namely that it solves the problem

$$\max\nolimits_{\{k_t\}_{t\geq 1}} \sum\nolimits_{t\geq 1} \beta^t V(k_t)$$

$$\text{subject to}\ \ \sum\nolimits_{t\geq s} \beta^{t\text{-}s}\, V(k_t) - D(k_s) \geq 0,\ \ s \geq 1\,.$$

To this end observe that to improve upon the choice $k_t = k^*\ \forall t \geq 1$ one has to raise at least one k_t from k^*. We show that this cannot be done without violating some constraint (keep in mind that if $k_t = k^* \ \forall t \geq 1$ all the constraints hold with equality). Without loss of generality suppose k_1 is raised, say to $k^* + h_1$. By definition of k^*, cfr. equation (8), it will be $\Delta D > (1-\beta)^{-1}\Delta V$, so if one keeps $k_t = k^* \ \forall t \geq 2$, since

$$\Delta\left(\sum\nolimits_{t\geq 1}\beta^{t-1}V(k_t)\right) = \Delta V < (1-\beta)\Delta D < \Delta D$$

the constraint at $s = 1$ is violated (Δ refers here to raising k_1 from k^* to $k^* + h_1$ of course); hence to restore it one should raise k_2 —or some other k_t $t \geq 2$, the argument does not change. But by the same token, if one raises k_2 one then has to raise k_3 (or $k_{t_3}\ldots$) to restore the ($s = 2$)–constraint, and so on: that is, if k_1 is raised from k^* one should keep raising k's farther and farther away. Can this be done ad infinitum? The question here is, by how much does k_2 need to be raised to restore feasibility at $s = 1$? From the above displayed inequalities it follows that the needed increment of k_2 would be larger than the increment needed if the first inequality there were instead an equality, i.e. the h_2 such that

$$\beta\big(V(k^*+h_2) - V(k^*)\big) = \beta\big(D(k^*+h_1) - D(k^*)\big).$$

But $V(k^*+h_2) - V(k^*) < V'(k^*)h_2$, $D(k^*+h_1) - D(k^*) > D'(k^*)h_1$, and from (8) one has $V'(k^*) < (1-\beta)D'(k^*)$; therefore

$$h_2 > \frac{D'(k^*)}{V'(k^*)}\,h_1 > (1-\beta)^{-1}\,h_1 > h_1.$$

Analogously, to restore feasibility at $t = 2$ one should then have to raise k_3 by an amount $h_3 > h_2$, and by so doing it is clear that one hits the upper bound K in a finite number of steps. The conclusion is that it is in fact impossible to improve upon the choice $k_t = k^* \ \forall t \geq 2$, as was to be shown.

Proof of Proposition 3. We first put on record an observation:

Lemma. *Fix a time τ, and assume $k_t = K_t \ \forall t > \tau$. Then all incentive constraints for $s > \tau$ are satisfied with equality if $\phi_t = \hat{\phi}_t \ \forall t > \tau$.*

Proof. Recall that by assumption (9) $V_t(K_t) = V_0(K_0)$, which in turn implies $V_t(K_t) = (1-\beta)\mathcal{V} \ \forall t$. Then for $s > \tau$, given that $k_t = K_t$ and $\phi_t = \hat{\phi}_t$ for $t \geq s$, the incentive constraint at s is $(1-\beta)\mathcal{V}\sum_{t\geq s}\beta^{t-s}\hat{\phi}_t \leq \mathcal{V} - D_s(K_s)$, which, by assumption (10) and the fact that $\hat{\phi}_t = \gamma^{(s-\tau)+(t-s)}\hat{\phi}_\tau$, can be written as

$$\frac{(1-\beta)\mathcal{V}}{1-\beta\gamma}\,\gamma^{s-\tau}\hat{\phi}_\tau \leq \gamma^s\big(\mathcal{V} - D_0(k_0)\big).$$

We just have to plug in the definition of $\hat{\phi}_\tau$, page 8 to verify that equality holds. □

Consider now the case $t_0 = 0$, i.e. $\hat{\phi}_0 \leq 1$. Start again from observing that the incentive constraint at $t = 0$ has the objective function on the left. As before, try to maximize the right member and subject to have equality in the constraint. Since the right member is $\sum_{t\geq 0}\beta^t\,V_0(k_t) - D_0(k_0)$, set first $k_t = K_t \ \forall t \geq 1$, and then $\phi_t = \hat{\phi}_t \ \forall t \geq 1$ to have the other constraints satisfied (with equality, from the lemma). This way the constraint at zero becomes

$$\phi_0 V_0(k_0) + \beta\big(\mathcal{V} - D_1(K_1)\big) \leq V_0(k_0) - D_0(k_0) + \beta\mathcal{V}.$$

If we set $k_0 = K_0$ and $\phi_0 = \hat{\phi}_0$ we have equality by definition, so from $\hat{\phi}_0 \leq 1$,

$$V_0(K_0) + \beta\big(\mathcal{V} - D_1(K_1)\big) \geq V_0(K_0) - D_0(K_0) + \beta\mathcal{V}. \tag{15}$$

Suppose first that $\hat{\phi}_0 < 1$, so that the above inequality is strict; if we lower k_0 from K_0 towards $\operatorname{argmax}(V_0 - D_0)$ the right member goes up, the left one down, and two possibilities arise: (i) equality is reached at some $k^* \in \big(\operatorname{argmax}(V_0 - D_0), K_0\big)$;

in this case the value $V_0(k^*) - D_0(k^*) + \beta\mathcal{V}$ is the highest possible principal's payoff, attainable with $\phi_0 = 1$ (if we lower k_0 further the left member, i.e. the principal's payoff, decreases, with $\phi_0 = 1$ and even more for any $\phi_0 < 1$); thus optimal policy is here $k_0 = k^*$ (defined by the equality $D_0(k) = \beta D_1(K_1)$), $k_t = K_t\ \forall t \geq 1,\ \phi_0 = 1,\ \phi_t = \hat{\phi}_t\ \forall t \geq 1$; (ii) at argmax$(V_0 - D_0)$ inequality in (15) is still strict; in this case the maximum possible value of $\sum_{t\geq 0} \beta^t\, V_t(k_t) - D_0(k_0)$, i.e. $\max\big[V_0 - D_0\big] + \beta\mathcal{V}$, is attainable with the $\phi_0 < 1$ defined by $\phi_0 V(\text{argmax}(V_0 - D_0)) + \beta\big(\mathcal{V} - D_1(K_1)\big) = \max\,[V_0 - D_0] + \beta\mathcal{V}$, and optimal policy has the ϕ_0 just defined, $k_0 = \text{argmax}\,[V_0 - D_0]$, and continuation for $t \geq 1$ as in the previous case.

If on the other hand $\hat{\phi}_0 = 1$, so (15) is an equality, then lowering k_0 from K_0 can only do harm (period–zero incentive constraint would hold with strict inequality, and the left member, i.e. the principal's payoff, would be lower than with $k_0 = K_0$). Hence optimal poicy in this case is $k_t = K_t\ \forall t \geq 0,\ \phi_0 = 1,\ \phi_t = \hat{\phi}_t\ \forall t \geq 1$. This ends the case $t_0 = 0$.

We now turn to the case $t_0 > 0$; recall that this means $\hat{\phi}_{t_0} \equiv \gamma^{t_0}(1 - \beta\gamma)(\mathcal{V} - D_0(K_0))/(1 - \beta)\mathcal{V} \leq 1$, but $\hat{\phi}_t > 1\ \forall t < t_0$. Let us write the principal's payoff as

$$\sum\nolimits_{t=0}^{t_0-1} \beta^t \phi_t V_t(k_t) + \beta^{t_0} \sum\nolimits_{t\geq t_0} \beta^{t-t_0} \phi_t V_t(k_t). \tag{16}$$

Taking into account the constraints for $t \geq t_0$, which still have to be met, we know from the previous case what the constrained maximum of the second sum is, and the policy which achieves it.

Suppose first that $\hat{\phi}_{t_0} = 1$. Then if $k_{t_0} = K_{t_0}$, the incentive constraints for $t \geq t_0$ are all satisfied with equality (those for $t > t_{t_0}$ from the lemma, the one at t_0 checked easily). Moreover, in this case we shall now verify that it is feasible to set $k_t = K_t,\ \phi_t = 1\ \forall t < t_0$; since this is the best one can hope for, optimal policy is found: $k_t = K_t\ \forall t$, $\phi_t = 1\ \forall t \leq t_0$ and $\phi_t = \hat{\phi}_t\ \forall t > t_0$. To verify feasibility of the policy for $t < t_0$ consider the constraint at $t_0 - 1$, which with the given policy becomes

$$V_{t_0-1}(K_{t_0-1}) + \beta \sum\nolimits_{t\geq t_0} \beta^{t-t_0} \phi_t V_t(K_t) \leq \mathcal{V} - D_{t_0-1}(K_{t_0-1});$$

but $\sum_{t\geq t_0} \beta^{t-t_0}\phi_t V_t(K_t) = \mathcal{V} - D_{t_0}(K_{t_0}) = \gamma^{t_0}(\mathcal{V} - D_0(K_0))$, so the left member is equal to $(1-\beta)\mathcal{V} + \beta\gamma^{t_0}(\mathcal{V} - D_0)$; and since the right member is $\gamma^{t_0-1}(\mathcal{V} - D_0(K_{t_0}))$, the constraint is $(1 - \beta)\mathcal{V} + \beta\gamma^{t_0}(\mathcal{V} - D_0(K_0)) \leq \gamma^{t_0-1}(\mathcal{V} - D_0(K_{t_0}))$; rearranging, this is just $\hat{\phi}_{t_0-1} \geq 1$, true by hypothesis. Analogously, the (t_0–2)–constraint becomes $\hat{\phi}_{t_0-2} \geq 1$, and so on down to zero.

Consider now the case $\hat{\phi}_{t_0} < 1$. Here as we know the policy maximizing the second sum in (16) calls for $k_{t_0} < K_{t_0}$, and this creates a trade–off: for a lower k_{t_0} implies a lower V_{t_0}, and this in turn tightens the incentive constraints for $t < t_0$. For example, it makes the policy of full transfer knowledge and full appropriation for $t < t_0$, found optimal just above when $\hat{\phi}_{t_0} = 1$, generally unfeasible. Given that optimal policy for $t > t_0$ remains the one defined before ($k_t = K_t,\ \phi_t = \hat{\phi}_t$), choice for $t \leq t_0$ solves the finite–dimensional problem just introduced, of maximizing $\sum_{t=0}^{t_0} \beta^t \phi_t V_t(k_t)$ subject to the constraints for $t \leq t_0$ (the values of k_t, ϕ_t for $t \geq t_0$ being fixed). This ends the proof.

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Salvatore Modica: Facoltà di Economia, Università di Palermo