Integrali e Serie di Potenze

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Integrali

1. $\displaystyle\int \frac{1+\cos x}{x+\sin x}dx$
2. $\displaystyle\int \frac{3x+2}{x^2+1}dx$
3. $\displaystyle\int \frac{1}{1-\sin^2 x}dx$
4. $\displaystyle\int \arcsin x dx$
5. $\displaystyle\int x^2 \log^2 x dx$
6. $\displaystyle\int_0^{\frac{\pi}{2}} \sin(x)\cos^5(x)dx$
7. $\displaystyle\int \frac{dx}{x\log^3 x}$
8. $\displaystyle\int \frac{2\sin x\cos x}{1+\sin^2 x}$
9. $\displaystyle\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$ (porre $t = x^6$)
10. $\displaystyle\int \frac{x^6-5x^5+2x^3+1}{4x^5}dx$
11. $\displaystyle\int \frac{x^3+5x^2+4x+4}{x^2+1}dx$
12. $\displaystyle\int_{e-1}^{e^2-1} \frac{1}{(x+1)\log(x+1)}dx$

Soluzioni

1. $\log|x+\sin x|+c$
2. $\frac{3}{2}\log(x^2+1)+2\arctan x+c$
3. $\tan x+c$
4. $x\arcsin x+\sqrt{1-x^2}+c$
5. $\frac{1}{3}x^3\left(\log^2 x-\frac{2}{3}\log x+\frac{2}{9}\right)+c$
6. $\frac{2}{3}$ $\left[\text{La primitiva: } -\frac{1}{6}\cos^6 x+c\right]$
7. $-\frac{1}{2\log^2 x}+c$
8. $\log(1+\sin^2 x)+c$
9. $2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log(\sqrt[6]{x}+1)+c$
10. $\frac{x^2}{8}-\frac{1}{2x}-\frac{5}{4}x-\frac{1}{16x^4}+c$
11. $\frac{x^2}{2}+5x+\frac{3}{2}\log(x^2+1)-\arctan x+c$
12. $-\log(\log(x+1))+c$

Serie di potenze

1. $\displaystyle\sum_{n=1}^{+\infty}(-1)^n\frac{9^n-5^n}{(4n+7)8^2}\left(x-\frac{1}{2}\right)^n$
2. $\displaystyle\sum_{n=1}^{+\infty}\frac{n!2^n+5}{(n+3)!}(x+2)^n$
3. $\displaystyle\sum_{n=1}^{+\infty}\frac{3^n}{2^{2n+1}}(x-1)^n$
4. $\displaystyle\sum_{n=1}^{+\infty}\frac{n^2}{n!2^n}x^n$
5. $\displaystyle\sum_{n=1}^{+\infty}\frac{4^n}{n^4-n-4}\left(x-\frac{1}{4}\right)^n$
6. $\displaystyle\sum_{n=1}^{+\infty}(-1)^n\frac{(x-3)^n}{n3^n}$

Soluzioni

1. $x\in\left(-\frac{7}{18},\frac{25}{18}\right)$
2. $x\in\left(-\frac{5}{2},-\frac{3}{2}\right)$
3. $x\in\left(\frac{1}{4},\frac{7}{4}\right)$
4. $x\in\mathbb{R}$
5. $x\in\left(0,\frac{1}{2}\right)$
6. $x\in(0,6)$