Università degli Studi di Palermo

Dipartimento di Matematica e Informatica

Words and Automata Research Group

SEMINAR ANNOUNCEMENT

Title: Periodicity of Words and Measures of Complexity

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Room 7
Via Archirafi 34, 90123 Palermo

Abstract:

One of the most important results in Combinatorics on Words is the Critical Factorization Theorem (CFT), which establishes a relationship between "local" periodicities of a *finite* word and its global periodicity. The starting point of this research is the extension of the CFT to infinite words.
Given an infinite word **x**, one defines the *periodicity function* $p_{\mathbf{x}}(n)$, which, for every position n, gives the "local period" at n, i.e. the length of the shortest "square" centered in position n. By extending the CFT to infinite words, one can prove that an infinite word **x** is periodic if and only if its periodicity function $p_{\mathbf{x}}(n)$ is bounded. One can also prove a *gap theorem* for the function $p_{\mathbf{x}}(n)$: either $p_{\mathbf{x}}(n)$ is bounded, or $p_{\mathbf{x}}(n) \geq n+1$ for infinitely many values of n.
The first result of this research provides a characterizations of the *characteristic Sturmian words* in terms of the periodicity function: we prove that an infinite word **x** is a *characteristic Sturmian word* if and only if $p_{\mathbf{x}}(n) \leq n+1$ for every $n \geq 1$, and is equal to n+1 for infinitely many values of n. In this sense, the *characteristic Sturmian words* correspond to the extremal case of the CFT for infinite words.
The previous result suggests that infinite words could be studied (for example, classified) through their periodicity function, considering it as a new measure of complexity. Nevertheless, the periodicity function has strong fluctuations, and does not seem adequate as a measure of complexity. Therefore, we introduce, for every infinite word **x**, the function $h_{\mathbf{x}}(n)$, which gives the *mean value* of the periodicity function $p_{\mathbf{x}}(n)$ between 1 and n. The function $h_{\mathbf{x}}(n)$ is called the *periodicity complexity function* of the word **x**, and results in a useful tool for studying infinite words.

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The result we present here show that this new measure of complexity is independent from other measures of complexity commonly used in Combinatorics on Words, as for example the *factor complexity*. In particular, we show the following results:

1. There exist non-periodic recurrent infinite words **x** such that $h_{\mathbf{x}}$ is bounded.
2. If **f** is the Fibonacci word, then $h_{\mathbf{f}}$ has logarithmic growth.
3. If **t** is the Thue-Morse word, then $h_{\mathbf{t}}$ has linear growth.
4. There exist recurrent infinite words **x** such that $h_{\mathbf{x}}$ has arbitrary growth.

These results show the interest of this new measure of complexity in the study of infinite words, and outline possible further developments of this research.

All interested people, in particular students, are invited to participate.

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