On K-ary n-Cubes ad Isometric Words*

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A k-ary n-cube is a graph with k^n vertices, each associated to a word of length n over an alphabet of cardinality k. The subgraph obtained deleting those vertices which contain a given k-ary word f as a factor is called the *k-ary n-cube avoiding f*. When, for any n, such a subgraph is *isometric* to the cube, the word f is said *isometric* (or *good*). In the binary case, isometric words can be equivalently defined, independently from hypercubes. Recently, this problem has been investigated in the binary case [4]. These two approaches are here considered and extended to the k-ary alphabet, showing that they are still coincident for $k = 3$, also for $k = 4$, not with Hamming distance but with Lee distance, and they are not still coincident from $k > 4$ on. [1, 2]. A word f is also called *d-good* if for any pair of words u and v of length d, with $d \geq |f|$, which do not contain the factor f, u can be transformed in v by exchanging one by one the symbols on which they differ and generating only words which do not contain f. It is *good* if it is d-good for all $d \geq |f|$, it is *bad* if it is not good. The *index* of a bad word is the threshold d from which the word is no longer d-good, and a pair of words (u, v) showing that the word is not good is called a pair of *witnesses* for the bad word. A characterization of these k-ary words is here provided also in terms of 2-*error-overlap*. Furthermore, here is an algorithm to decide whether a word is bad or not. When the word is bad the algorithm provides its index and a pair of witnesses of minimal length. An analogous algorithm is given for binary alphabet in [4]; it runs in cubic time. Our algorithm works on k-ary words, with $k \leq 4$ and construct an enhanced suffix tree of the word. This data structure has been used in [3] to design an algorithm to decide whether a k-ary word is "bad" or not (without computing the index and the witnesses). Algorithm here provided runs in $O(n)$ time with a preprocessing of $O(n)$ time and space.

References

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